

MATH 0200 - Prep for Scientific Calculus
SAMPLE FINAL EXAM 2

Exam length: 1 hour 50 minutes

1. (30 points)

Simplify or Solve.

- (a) Find all x such that $\frac{x+2}{x-1} < 3$.
- (b) Give exact value for $\cos(\pi/12)$.
- (c) Give exact value for $\sin(7\pi/12)$.
- (d) Evaluate $\cos^{-1}(\cos(10\pi/9))$.
- (e) Evaluate $\cos(\tan^{-1}(-4))$.
- (f) Find all real and/or complex solutions to $z^2 + 4z + 6 = 0$.

2. (30 points)

Short answer.

- (a) The graph of $g(x)$ is obtained from the graph of $f(x)$ by first stretching the graph of f horizontally by a factor of 2, then flipping it vertically (across the x -axis), then shifting it up 2 units. Give a formula for $g(x)$ in terms of $f(x)$.
- (b) Sketch the graph of $f(x) = \sqrt{x}$ and its inverse. Clearly mark the domains and ranges of each.
- (c) Give the equation of the line perpendicular to $y = 3x + 5$ and through the point $(1, 2)$.
- (d) What is the minimum value of $x^2 - 6x + 2$?
- (e) You have a calculator which can only compute square roots and multiply numbers. Explain how you can use it to compute $7^{3/4}$.
- (f) Find a number b such that 3 is a zero of $p(x) = 1 - 4x + bx^2 + 2x^3$.

3. (30 points)

Short answer.

- (a) List all asymptotes of $f(x) = \frac{x^2 + 1}{(x-1)(x+2)}$.
- (b) Find a number n such that $\log_3(\log_2 n) = 2$.
- (c) Write as a single logarithm: $\log x + 2 \log y - \frac{1}{2} \log z^4$.
- (d) Write an expression for the amount of money you will have after t years if you invest \$1000 at a rate of 2% compounded quarterly.
- (e) What is the slope of the line that makes an angle of 30° with the positive x -axis?
- (f) Find two angles u and v so that $\cos u = \cos v$ but $\sin u \neq \sin v$.

4. (10 points)

Show that if $f(x) = mx + b$ then for $m \neq 0$ its inverse is given by the equation

$$f^{-1}(y) = \frac{1}{m}y - \frac{b}{m}.$$

5. (15 points)

For the functions

$$r(x) = \frac{3x + 4}{x^2 + 1}, \quad s(x) = \frac{x^2 + 2}{2x - 1}$$

find and SIMPLIFY the formula for the composition $r \circ s(x)$.

6. (15 points)

Solve

$$\frac{\ln(12x)}{\ln(5x)} = 2.$$

7. (15 points)

Simplify $\left(\frac{4}{5} - \frac{3}{5}i\right)^{75}$.

8. (25 points)

Suppose θ is in the interval $(\pi/2, \pi)$ with $\sin \theta = 1/3$. Compute:

(a) $\cos \theta$

(b) $\tan \theta$

(c) $\sec \theta$

(d) $\csc \theta$

(e) $\cot \theta$

(f) $\sin(\theta + \pi)$

(g) $\cos(\theta + \pi)$

(h) $\sin(\pi/2 - \theta)$

(i) $\cos(\pi/2 - \theta)$

(j) $\sin(-\theta)$

(k) $\cos(-\theta)$

(l) $\sin(\theta/2)$

(m) $\cos(2\theta)$

9. (15 points)

Find the area of a regular hexagon inscribed in the unit circle.

10. (15 points)

Find all solutions to $z^3 = -8i$.