MATH 0200 - Prep for Scientific Calculus
SAMPLE FINAL EXAM 1

Exam length: 1 hour 50 minutes

1. (30 points)
Short answer. Fill in the blank.

(a) Function $f(x) = 6x^3 + 7x - 3$ has degree ____________.

(b) State the Law of Cosines and sketch the corresponding triangle.

(c) A function with vertical asymptotes at $x = 2$ and $x = 3$ and horizontal asymptote $y = 7$ is ______________________.

(d) $\tan\left(\frac{\pi}{2} - \theta\right) =$ ____________(θ).

(e) The conic section
$$x^2 + 4x + 25y^2 - 50y = -4$$
is a ______________.

(f) The equation of the line parallel to $y = -x - 16$ and through the point $(3, 2)$ is ________________________.

2. (30 points)
Short answer. Fill in the blank.

(a) $27° =$ ____________ rad.

(b) Graph of $2f(x-1)$ can be obtained from the graph of $f(x)$ by ___________________ in the ___________ direction and ___________________ in the ___________ direction. Answers should be shifting/scaling/flipping and vertical/horizontal.

(c) The period of $f(x) = \sin x$ is _________. The period of $\cos(2x)$ is _________.

(d) The range of $\cos^{-1}(x)$ is _____________. The range of $\sin^{-1}(x)$ is _____________.

(e) The polar equation for the circle centered at the origin with radius 3 is _____________.

(f) Write the equation for the balance in a bank account after t years with annual interest rate of 7% compounded continuously and initial deposit $15,000. ______________________

3. (30 points)
Solve:

(a) $|-2x + 1| - 5 = 10$

(b) $e^{2x} + 2 = 8 - e^x$

(c) $\tan\left(\frac{7\pi}{6}\right)$

(d) $\tan\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$

(e) $\sin\left(-\sin^{-1}\left(\frac{3}{13}\right)\right)$

(f) $\log_2(x + 3) + \log_2(x - 2) = 3$

4. (30 points)
Simplify:

(a) $27^{-4/3}$

(b) $\left(\dfrac{x^3y^2 + xy^{-2}}{x^{-3}y^{-4}}\right)^{-1}$

(c) $\ln\left(\dfrac{x^3}{y^2}\right)$ if $\ln x = 2$ and $\ln y = 6.1$

(d) $\dfrac{3-4i}{6-5i}$

(e) $\left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right)^{300}$

5. (15 points)
Let

$$f(x) = \frac{x}{x-2}, \qquad g(x) = \sqrt{x+1}.$$

(a) What is the domain of $g \circ f(x)$?

(b) Evaluate $g \circ f(3)$.

(c) Write an expression for $f^{-1}(x)$.

6. (10 points)
Find a number t such that the vectors $\mathbf{u} = \langle 2\cos t, 4\rangle$ and $\mathbf{v} = \langle 10, 3\rangle$ are perpendicular.

7. (10 points)
Suppose a 40-foot ladder is leaning against a a wall, making a 45° angle with the ground (measured from a perpendicular line from the base of the ladder to the wall).

(a) How high up the wall is the end of the ladder?

(b) How far from the wall is the base of the ladder?

8. (15 points)
Suppose u and v are angle in the interval $(0, \pi/2)$ with $\cos u = 1/3$ and $\sin v = 1/4$. Evaluate:

(a) $\sin u$

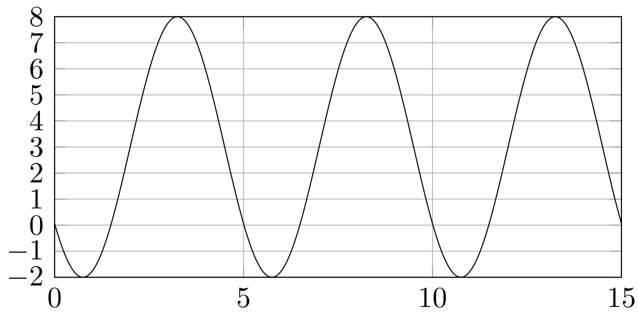
(b) $\cos v$

(c) $\sin(2u)$

(d) $\cos(2v)$

(e) $\cos(u/2)$

9. (15 points)
Find a function which models the periodic behavior. The first minimum occurs at $x = 3/4$.



10. (15 points)
Determine the area of a regular octagon with vertices on the unit circle.