

Math 0240 - Analytic Geometry & Calculus 3

Final Exam, Fall 2016

INSTRUCTIONS:

- No calculators, electronic devices, notes, books or memory cards are allowed.
 - Please write legibly and logically, and show all work. **Incomplete explanations may receive little or no credit.**
 - **Warning:** You are expected to work independently.
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There are ten problems for a total of 100 points.

1 _____(10) 2 _____(10) 3 _____(10) 4 _____(10) 5 _____(10)

6 _____(10) 7 _____(10) 8 _____(10) 9 _____(10) 10 _____(10)

TOTAL _____ (100)

1. (10pts) Let C be the curve given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 3e^{t-1}\mathbf{k}$. At the point $P(1, 1, 3)$, find the curvature of C and parametric or symmetric equations of the line tangent to C .
2. (10pts) Let $f(x, y, z) = \frac{x-y}{z} + 4\sqrt{x+3z}$ and P be the point $P(1, 1, 1)$. The following three parts are relevant. You do not need to repeat any calculation.
 - (a) (4pts) What is the direction in which the maximum rate of change of f occurs at the point P ?
 - (b) (3pts) Compute the directional derivative of $f(x, y, z)$ at the point P in the direction of the vector $v = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.
 - (c) (3pts) At the point $P(1, 1, 1)$, the equation $\frac{x-y}{z} + 4\sqrt{x+3z} = 8$ holds. Use the Implicit Function Theorem to find $z_x(1, 1)$.
3. (10pts) Let S be the ellipsoid given by the equation $x^2 + y^2 - xz + z^2 = 2$. That is, S is a level surface of the function $F(x, y, z) = x^2 + y^2 - xz + z^2$. Find all points on S where the tangent plane is parallel to the plane $x + 2y + z = 10$. (Hint: Use the fact that the coordinates of such a point satisfy the equation of S .)
4. (10pts) Find all critical points of the function $f(x, y) = 2x^2 + y^2 - x^2y$. For each critical point determine if it is a local maximum, a local minimum, or a saddle point.

5. (10pts) Suppose that the volume of a solid E can be represented by the triple integral

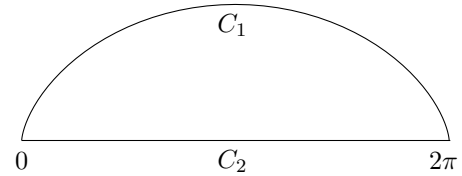
$$\iiint_E dV = \int_0^{\frac{\sqrt{2}}{2}} \int_0^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz dy dx.$$

Find the mass of the solid E , if the density function is given by $\rho(x, y, z) = e^{(x^2+y^2+z^2)^{3/2}}$.

6. (10pts) Given the vector field $\mathbf{F}(x, y) = (2x \ln y - y)\mathbf{i} + (x^2y^{-1} - x)\mathbf{j}$ defined on $\{(x, y) \mid y > 0\}$.
 - (a) (6pts) Show that \mathbf{F} is conservative and find a potential function f .
 - (b) (4pts) A particle, under the influence of the vector field \mathbf{F} , moves along the curve C given by $\mathbf{r}(t) = (3t)\mathbf{i} + (2t^2 + 1)\mathbf{j}$ from $t = 0$ to $t = 1$. Use the Fundamental Theorem of line integrals to find the work done.

7. (10pts)

- (a) (4pts) Use Green's Theorem to show that a region R enclosed by a simple closed curve C , oriented clockwise, has area $\int_C y dx$.
- (b) (6pts) Use part a) to compute the area of the region D enclosed by the arch of the cycloid $C_1 : x = t - \sin t, y = 1 - \cos t$ from $(0,0)$ to $(2\pi,0)$ and the line segment $C_2 : x = t, y = 0$ from $(2\pi,0)$ to $(0,0)$. See the sketch below.



8. (10pts) Find the area of the surface S that is the part of the cylinder $x^2 + y^2 = 1$, below the plane $z = 3 - x - y$ and above the plane $z = 0$.

9. (10pts) Use Stoke's Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x^2y \mathbf{i} + \frac{1}{3}x^3 \mathbf{j} + xy \mathbf{k}$, and where C is the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the hyperbolic paraboloid $z = y^2 - x^2$, oriented counterclockwise when viewed from above.

10. (10pts) Let S be the boundary surface of the solid E enclosed by the paraboloids $z = 1 + x^2 + y^2$ and $z = 2(x^2 + y^2)$, with the normal pointing outward. Compute the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (e^{\sin z} - x^2) \mathbf{i} + 2xy \mathbf{j} + (z^2 - \cos y) \mathbf{k}$.