Math 0240 - Analytic Geometry & Calculus 3
Final Exam, Fall 2016

INSTRUCTIONS:

- **No calculators, electronic devices, notes, books or memory cards are allowed.**
- Please write legibly and logically, and show all work. **Incomplete explanations may receive little or no credit.**
- **Warning:** You are expected to work independently.

There are ten problems for a total of 100 points.

1______(10)	**2**______(10)	**3**______(10)	**4**______(10)	**5**______(10)
6______(10)	**7**______(10)	**8**______(10)	**9**______(10)	**10**______(10)

TOTAL____________ (100)

1. (10pts) Let C be the curve given by $\mathbf{r}(t) = t\,\mathbf{i} + t^2\mathbf{j} + 3e^{t-1}\mathbf{k}$. At the point $P(1,1,3)$, find the curvature of C and parametric or symmetric equations of the line tangent to C.

2. (10pts) Let $f(x,y,z) = \dfrac{x-y}{z} + 4\sqrt{x+3z}$ and P be the point $P(1,1,1)$. The following three parts are relevant. You do not need to repeat any calculation.
   (a) (4pts) What is the direction in which the maximum rate of change of f occurs at the point P?
   (b) (3pts) Compute the directional derivative of $f(x,y,z)$ at the point P in the direction of the vector $v = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.
   (c) (3pts) At the point $P(1,1,1)$, the equation $\dfrac{x-y}{z} + 4\sqrt{x+3z} = 8$ holds. Use the Implicit Function Theorem to find $z_x(1,1)$.

3. (10pts) Let S be the ellipsoid given by the equation $x^2 + y^2 - xz + z^2 = 2$. That is, S is a level surface of the function $F(x,y,z) = x^2 + y^2 - xz + z^2$. Find all points on S where the tangent plane is parallel to the plane $x + 2y + z = 10$. (Hint: Use the fact that the coordinates of such a point satisfy the equation of S.)

4. (10pts) Find all critical points of the function $f(x,y) = 2x^2 + y^2 - x^2y$. For each critical point determine if it is a local maximum, a local minimum, or a saddle point.

5. (10pts) Suppose that the volume of a solid E can be represented by the triple integral

$$\iiint_E dV = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz\,dy\,dx.$$

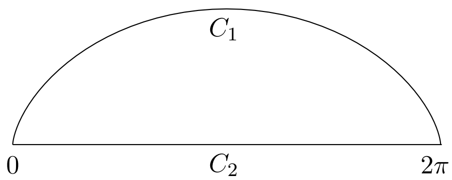
   Find the mass of the solid E, if the density function is given by $\rho(x,y,z) = e^{(x^2+y^2+z^2)^{3/2}}$.

6. (10pts) Given the vector field $\mathbf{F}(x,y) = (2x\ln y - y)\,\mathbf{i} + \left(x^2y^{-1} - x\right)\,\mathbf{j}$ defined on $\{(x,y) \mid y > 0\}$.
   (a) (6pts) Show that $\mathbf{F}$ is conservative and find a potential function f.
   (b) (4pts) A particle, under the influence of the vector field $\mathbf{F}$, moves along the curve C given by $\mathbf{r}(t) = (3t)\mathbf{i} + (2t^2+1)\mathbf{j}$ from $t = 0$ to $t = 1$. Use the Fundamental Theorem of line integrals to find the work done.

7. (10pts)

   (a) (4pts) Use Green's Theorem to show that a region R enclosed by a simple closed curve C, oriented clockwise, has area $\int_C y\,dx$.

   (b) (6pts) Use part a) to compute the area of the region D enclosed by the arch of the cycloid $C_1:\ x = t - \sin t,\ y = 1 - \cos t$ from $(0,0)$ to $(2\pi, 0)$ and the line segment $C_2:\ x = t,\ y = 0$ from $(2\pi, 0)$ to $(0,0)$. See the sketch below.



8. (10pts) Find the area of the surface S that is the part of the cylinder $x^2 + y^2 = 1$, below the plane $z = 3 - x - y$ and above the plane $z = 0$.

9. (10pts) Use Stoke's Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z) = x^2 y\,\mathbf{i} + \frac{1}{3}x^3\,\mathbf{j} + xy\,\mathbf{k}$, and where C is the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the hyperbolic paraboloid $z = y^2 - x^2$, oriented counterclockwise when viewed from above.

10. (10pts) Let S be the boundary surface of the solid E enclosed by the paraboloids $z = 1 + x^2 + y^2$ and $z = 2(x^2 + y^2)$, with the normal pointing outward. Compute the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = \left(e^{\sin z} - x^2\right)\mathbf{i} + 2xy\mathbf{j} + (z^2 - \cos y)\mathbf{k}$.