MATH 0240 - FINAL EXAM
CALCULUS III

Monday, April 22, 2013 - 2:00 - 3:50 a.m.

LAST NAME: ____________________ FIRST NAME: ____________________

SIGNATURE:________________________________

PEOPLE SOFT NO #:____________________________

CIRCLE YOUR INSTRUCTOR: CONSTANTINE HAHN SYSOEVA

DIRECTIONS:

1. Show all work for partial credit.
2. No CALCULATORS permitted.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (a) (5 pts.) Find the unit tangent vector $\overrightarrow{T}$ and unit normal vector $\overrightarrow{N}$ to the curve $\overrightarrow{r}(t) = \langle 3\cos t, 4t, 3\sin t\rangle$ at the point

$$P = (\frac{-3}{\sqrt{2}},\ 3\pi,\ \frac{3}{\sqrt{2}})$$

(b) (5 pts.) Find the curvature of the curve at the point P.

2. Use linear approximation to approximate the number:

$$\sqrt{3.04 + e^{-0.08}}$$

3. Determine all local maxima, local minima, and saddle points of $f(x, y) = 3y - y^3 - 3x^2y$.

4. A rectangular box without a lid is to be made from 48 ft^2 of cardboard. Find the maximum volume of the box. (**Hint:** Use Lagrange multipliers).

5. Find the y coordinate of the center of mass of a lamina that occupies the region bounded by $y^2 = x + 4, x = 0$, and $y \geq 0$ and has density $\rho(x, y) = y$. Simplify your answer as much as possible.

6. Find the volume of the solid that lies within the cylinder $x^2 + y^2 = 4$, above the xy-plane, and below the cone $z^2 = 4x^2 + 4y^2$.

7. Let $\overrightarrow{F}$ be the two dimensional vector field given by $\overrightarrow{F}(x,y) = \langle ye^{xy} - 1, xe^{xy} + 2y \rangle$.

(a) Determine if $\overrightarrow{F}$ is a conservative vector field and If so find a potential function for $\overrightarrow{F}$.

(b) Find the value of the line integral $\int_C (\overrightarrow{F} \cdot \overrightarrow{T}) ds$ where C is the line segment from $(0,3)$ to $(5,0)$.

8. Use Green's Theorem to find the value of $\oint_C -5x^2dx + 7xy\ dy$ where C is the closed curve consisting of the edges of the triangle with vertices $(0,0)$, $(3,1)$ to $(0,3)$, oriented counterclockwise.

9. Use the Divergence Theorem to find the total flux $\int_S\int \overrightarrow{F} \cdot \overrightarrow{n}\, ds$, of the vector field $\overrightarrow{F} = \langle x^2, yz^2, -2xz \rangle$ across the surface S given by $x^2 + y^2 + z^2 = 2$ with outward orientation.

10. Use Stokes' Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = e^x \vec{i} + (x^2 + y^2)\vec{j} + z\vec{k}$ and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first quadrant oriented counterclockwise from above.