**MATH 0240 -Analytic Geometry and Calculus III**

**FINAL EXAMINATION**

**SPRING 2012**

NAME (Print): ______________________________

Student ID: ______________________________

NAME (Signature):______________________________

Please circle the name of your instructor:

Constantine Sysoeva

Please circle the time:

12:00 1:00

INSTRUCTIONS:

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, CELLPHONES OR COMPUTERS OR ANY OTHER ELECTRONIC DEVICES MAY BE USED.

2. Before you begin, Enter your Name and Student ID Number in the space above, Circle the Name of your Instructor and Meeting Time of your Lecture.

3. Show ALL of your Work on the Exam itself. If you need additional space, use the backs of the pages.

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1.______(10) 2.______(10) 3.______(10) 4.______(10) 5.______(10)

6.______(10) 7.______(10) 8.______(10) 9.______(10) 10.______(10)

TOTAL:_________(100)

**Problem 1.** Function $f$ is given by the formula $f(x,y) = 2x^2 + 3e^{xy}$.

a) Find the directional derivative of $f$ at the point $P = (1,0)$ in the direction of the vector $\mathbf{u} =< -1, 2 >$.

b) Find the maximal rate of change of $f(x,y)$ at $P$ and the direction in which it occurs.

**Problem 2.** The curve is given parametrically by

$$\mathbf{r}(t) = < t^3 + \frac{1}{2}t^2, 2t - 1, t^2 + t\sqrt{5} > .$$

a) Find its curvature at the point $(0, -1, 0)$.

b) Set up the integral representing the length of the curve from the point $(0, -1, 0)$ to the point $(10, 3, 4 + 2\sqrt{5})$.

DO NOT EVALUATE THE INTEGRAL.

**Problem 3.** Find an equation of the plane tangent to the surface

$$x^2 + y^2 z^2 = 8$$

at the point $P = (2, 2, 1)$.

**Problem 4.** Find all critical points of the function

$$f(x, y) = x^2 + 4xy - 10x + y^2 - 8y + 1.$$

For each critical point determine if it is a local maximum, a local minimum or a saddle point.

**Problem 5.** Find the work done by the force $\mathbf{F}(x, y) = 3y\mathbf{i} + x\mathbf{j}$ in moving a particle along the boundary of the trapeziod with the vertices $(0, 0)$, $(1, 1)$, $(2, 1)$ and $(3, 0)$ in the clockwise direction.

**Problem 6.** Find the mass of the solid bounded by the surfaces $y^2 + z^2 = 1$, $x = 0$ and $x = y^2 + z^2 - 4$, if the density function is given by the formula $\rho(x, y, z) = y^2 + z^2$.

**Problem 7.** a) Determine whether the vector field

$$\mathbf{F}(x, y, z) = (2y + 4z)\mathbf{i} + (2x + 3z)\mathbf{j} + (4x + 3y)\mathbf{k},$$

is conservative or not.

b) Evaluate

$$\int_C (2y + 4z)dx + (2x + 3z)dy + (4x + 3y)dz,$$

where $C$ is the curve given by

$$\mathbf{r}(t) =< t^3, 2\sin\left(\frac{\pi t}{2}\right), 3\cos\left(\frac{\pi t}{2}\right) >$$

for $0 \leq t \leq 1$.

**Problem 8.** Find the maximum and minimum values of the function $F(x, y, z) = x - y$ on the $x^2 + y^2 + xy + z^2 = 1$

**Problem 9.** Evaluate $\oint\limits_C \mathbf{F} \cdot d\mathbf{r}$,

if $\mathbf{F}(x, y, z) = y\mathbf{i} + 2x\mathbf{j} + yz\mathbf{k}$, and $C$ is the curve of intersection of the part of the paraboliod $z = 1 - x^2 - y^2$ in the first octant $(x \geq 0,\ y \geq 0,\ z \geq 0)$ with the coordinate planes $x = 0$, $y = 0$ and $z = 0$, oriented counterclockwise when viewed from above.

**Problem 10.** Evaluate

$$\iint_S \mathbf{F} \cdot \, d\mathbf{S},$$

if $\mathbf{F}(x, y, z) = (yz)\mathbf{i} + (x^2y)\mathbf{j} + (4zx^2)\mathbf{k}$ and $S$ is the surface of the solid bounded by the upper hemisphere $x^2 + y^2 + z^2 = 1, \ z \geq 0$, and the plane $z = 0$ with the normal pointing outward.