MATH 0240 - Analytic Geometry and Calculus III

FINAL EXAMINATION

FALL 2011

INSTRUCTIONS:

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.

2. Show ALL of your Work on the Exam itself.

1._____(10) 2._____(10) 3._____(10) 4._____(10) 5._____(10)

6._____(10) 7._____(10) 8._____(10) 9._____(10) 10._____(10)

TOTAL:________(100)

1. (a) (5 points) Find the unit tangent and unit normal vectors $\mathbf{T}$ and $\mathbf{N}$ to the curve

$$\mathbf{r}(t) = \langle 3\cos t,\ 4t,\ 3\sin t\rangle$$

at the point $P = \left(-\frac{3}{\sqrt{2}},\ 3\pi,\ \frac{3}{\sqrt{2}}\right)$.

(b) (5 points) Find curvature of the curve at the point P.

2. (10 points) Use LINEAR approximation to approximate the number $\sqrt{3.04 + e^{-0.08}}$.

3. (10 points) Find all critical points of the function $f(x, y) = 4x - 3x^3 - 2xy^2$. For each critical point determine if it is a local maximum, local minimum or a saddle point.

4. (10 points) Find the volume of the solid E bounded by $y = x^2$, $x = y^2$, $z = x + y + 5$, and $z = 0$.

5. (10 points) Find the y coordinate of the center of mass of a lamina that occupies the region bounded by $y^2 = x + 4$, $x = 0$, and $y \geq 0$ and has density $\rho(x, y) = y$. Simplify your answer as much as possible.

6. (10 points) Evaluate the integral

$$\iint\limits_R e^{x-2y}\, dA$$

where R is the parallelogram $ABCD$ with vertices $A = (0, 0)$, $B = (4, 1)$, $C = (7, 4)$, and $D = (3, 3)$ using the transformation $x = 4u + 3v$ and $y = u + 3v$. Simplify your answer as much as possible.

7. (10 points) Evaluate the line integral

$$\oint_C e^{2x+y}\,dx + e^{-y}\,dy$$

along the **negatively** oriented closed curve C, where C is the boundary of the triangle with the vertices $(0,0)$, $(0,1)$, and $(1,0)$.

8. (10 points) Evaluate the integral

$$\iint_S (10-2z)\,dS,$$

where S is the part of the surface $z = 5 - \dfrac{x^2}{2} - \dfrac{y^2}{2}$ inside the cylinder $x^2+y^2=1$.

9. (10 points) Evaluate the line integral

$$\oint_C \mathbf{F}\cdot d\mathbf{r}$$

for the vector field $\mathbf{F}(x,y,z) = -y\,\mathbf{i} + x\,\mathbf{j} - z\,\mathbf{k}$, where the closed curve C is the boundary of the triangle with vertices $(0,0,5)$, $(2,0,1)$, and $(0,3,2)$ traced in this order.

10. (10 points) Evaluate the flux of $\mathbf{F}(x,y,z) = z^2y\,\mathbf{i} + x^2y\,\mathbf{j} + (x+y)\,\mathbf{k}$ over S, where S is the closed surface consisting of the coordinate planes and the part of the sphere $x^2+y^2+z^2=4$ in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$, with the normal pointing outward.