

MATH 0200 - Prep for Scientific Calculus

SAMPLE FINAL EXAM

Exam length: 1 hour 50 minutes

INSTRUCTIONS:

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.
2. Show ALL of your calculations and display answers clearly. You may leave your final answers in exact form. Unjustified answers will receive no credit.
3. WRITE YOUR SOLUTIONS in the space provided. EXTRA SPACE is available on the BACKS of the pages. When using these back pages, clearly LABEL the problem, and also clearly indicate on the appropriate front page where your back-page solution (or continuation of a solution) is located.
4. Write neatly. Cross out any work that you do not wish to be considered for grading.
5. Academic Integrity Strictly Applies. Looking at another person's paper is reason to assume cheating and your paper will be taken.
6. All Cell phones and electronic devices must be OFF and put away and hats removed.

1. (30 points) Simplify or Solve.

(a) Find all  $x$  such that  $\frac{x+2}{x-1} < 3$ .

(b) Give exact value for  $\cos(\pi/12)$ .

(c) Give exact value for  $\sin(7\pi/12)$ .

(d) Evaluate  $\cos^{-1}(\cos(10\pi/9))$ .

(e) Evaluate  $\cos(\tan^{-1}(-4))$ .

(f) Find all real and/or complex solutions to  $z^2 + 4z + 6 = 0$ .

2. (30 points) Short answer.

(a) The graph of  $g(x)$  is obtained from the graph of  $f(x)$  by first stretching the graph of  $f$  horizontally by a factor of 2, then flipping it vertically (across the  $x$ -axis), then shifting it up 2 units. Give a formula for  $g(x)$  in terms of  $f(x)$ .

(b) Sketch the graph of  $f(x) = \sqrt{x}$  and its inverse. Clearly mark the domains and ranges of each.

(c) Give the equation of the line perpendicular to  $y = 3x + 5$  and through the point  $(1, 2)$ .

(d) What is the minimum value of  $x^2 - 6x + 2$ ?

(e) You have a calculator which can only compute square roots and multiply numbers. Explain how you can use it to compute  $7^{3/4}$ .

(f) Find a number  $b$  such that 3 is a zero of  $p(x) = 1 - 4x + bx^2 + 2x^3$ .

3. (30 points) Short answer.

(a) List all asymptotes of  $f(x) = \frac{x^2+1}{(x-1)(x+2)}$ .

(b) Find a number  $n$  such that  $\log_3(\log_2 n) = 2$ .

(c) Write as a single logarithm:  $\log x + 2 \log y - \frac{1}{2} \log z^4$ .

(d) Write an expression for the amount of money you will have after  $t$  years if you invest \$1000 at a rate of 2% compounded quarterly.

(e) What is the slope of the line that makes an angle of  $30^\circ$  with the positive  $x$ -axis?

(f) Find two angles  $u$  and  $v$  so that  $\cos u = \cos v$  but  $\sin u \neq \sin v$ .

4. (10 points) Show that if  $f(x) = mx + b$  then for  $m \neq 0$  its inverse is given by the equation  $f^{-1}(y) = \frac{1}{m}y - \frac{b}{m}$ .

5. (15 points) For the functions

$$r(x) = \frac{3x + 4}{x^2 + 1}, \quad s(x) = \frac{x^2 + 2}{2x - 1}$$

find and SIMPLIFY the formula for the composition  $r \circ s(x)$ .

6. (15 points) Solve

$$\frac{\ln(12x)}{\ln(5x)} = 2.$$

7. (15 points) Simplify  $(\frac{4}{5} - \frac{3}{5}i)^{75}$ .

8. (25 points) Suppose  $\theta$  is in the interval  $(\pi/2, \pi)$  with  $\sin \theta = 1/3$ . Compute:

(a)  $\cos \theta$  \_\_\_\_\_

(b)  $\tan \theta$  \_\_\_\_\_

(c)  $\sec \theta$  \_\_\_\_\_

(d)  $\csc \theta$  \_\_\_\_\_

(e)  $\cot \theta$  \_\_\_\_\_

(f)  $\sin(\theta + \pi)$  \_\_\_\_\_

(g)  $\cos(\theta + \pi)$  \_\_\_\_\_

(h)  $\sin(\pi/2 - \theta)$  \_\_\_\_\_

(i)  $\cos(\pi/2 - \theta)$  \_\_\_\_\_

(j)  $\sin(-\theta)$  \_\_\_\_\_

(k)  $\cos(-\theta)$  \_\_\_\_\_

(l)  $\sin(\theta/2)$  \_\_\_\_\_

(m)  $\cos(2\theta)$  \_\_\_\_\_

9. (15 points) Find the area of a regular hexagon inscribed in the unit circle.

10. (15 points) Find all solutions to  $z^3 = -8i$ .