MATH 0200 - Prep for Scientific Calculus

SAMPLE FINAL EXAM

Exam length: 1 hour 50 minutes

INSTRUCTIONS:

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.

2. Show ALL of your calculations and display answers clearly. You may leave your final answers in exact form. Unjustified answers will receive no credit.

3. WRITE YOUR SOLUTIONS in the space provided. EXTRA SPACE is available on the BACKS of the pages. When using these back pages, clearly LABEL the problem, and also clearly indicate on the appropriate front page where your back-page solution (or continuation of a solution) is located.

4. Write neatly. Cross out any work that you do not wish to be considered for grading.

5. Academic Integrity Strictly Applies. Looking at another person’s paper is reason to assume cheating and your paper will be taken.

6. All Cell phones and electronic devices must be OFF and put away and hats removed.
1. (30 points) Short answer. Fill in the blank.

(a) Function $f(x) = 6x^3 + 7x - 3$ has degree ________.

(b) State the Law of Cosines and sketch the corresponding triangle. ________________

(c) A function with vertical asymptotes at $x = 2$ and $x = 3$ and horizontal asymptote $y = 7$ is ________.

(d) $\tan(\pi/2 - \theta) = _______ (\theta)$.

(e) The conic section $x^2 + 4x + 25y^2 - 50y = -4$

is a ________.

(f) The equation of the line parallel to $y = -x - 16$ and through the point $(3, 2)$ is ________.
2. (30 points) Short answer. Fill in the blank.

(a) \(27^\circ = \underline{\quad} \text{ rad.}\)

(b) Graph of \(2f(x - 1)\) can be obtained from the graph of \(f(x)\) by \underline{\quad} in the \underline{\quad} direction and \underline{\quad} in the \underline{\quad} direction. Answers should be shifting/scaling/flippping and vertical/horizontal.

(c) The period of \(f(x) = \sin x\) is \underline{\quad}. The period of \(\cos(2x)\) is \underline{\quad}.

(d) The range of \(\cos^{-1}(x)\) is \underline{\quad}. The range of \(\sin^{-1}(x)\) is \underline{\quad}.

(e) The polar equation for the circle centered at the origin with radius 3 is \underline{\quad}.

(f) Write the equation for the balance in a bank account after \(t\) years with an annual interest rate of 7% compounded continuously and initial deposit \$15,000. \underline{\quad}.
3. (30 points) Solve:

(a) \(|-2x + 1| - 5 = 10\)

(b) \(e^{2x} + 2 = 8 - e^x\)

(c) \(\tan(7\pi/6)\)

(d) \(\tan(\cos^{-1}(1/3))\)

(e) \(\sin(-\sin^{-1}(3/13))\)

(f) \(\log_2(x + 3) + \log_2(x - 2) = 3\)
4. (30 points) Simplify:

(a) $27^{-4/3}$

(b) $\left(\frac{x^3y^2 + xy^{-2}}{x^{-3}y^{-4}}\right)^{-1}$

(c) $\ln\left(\frac{x^3}{y^2}\right)$ if $\ln x = 2$ and $\ln y = 6.1$

(d) $\frac{3-4i}{6-5i}$

(e) $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{300}$
5. (15 points) Let

\[ f(x) = \frac{x}{x - 2}, \quad g(x) = \sqrt{x + 1}. \]

(a) What is the domain of \( g \circ f(x) \)?

(b) Evaluate \( g \circ f(3) \).

(c) Write an expression for \( f^{-1}(x) \).

6. (10 points) Find a number \( t \) such that the vectors \( \mathbf{u} = \langle 2 \cos t, 4 \rangle \) and \( \mathbf{v} = \langle 10, 3 \rangle \) are perpendicular.
7. (10 points) Suppose a 40-foot ladder is leaning against a wall, making a 45\(^\circ\) angle with the ground (measured from a perpendicular line from the base of the ladder to the wall).

(a) How high up the wall is the end of the ladder?

(b) How far from the wall is the base of the ladder?

8. (15 points) Suppose \(u\) and \(v\) are angle in the interval \((0, \pi/2)\) with \(\cos u = 1/3\) and \(\sin v = 1/4\). Evaluate:

(a) \(\sin u\)

(b) \(\cos v\)

(c) \(\sin(2u)\)

(d) \(\cos(2v)\)

(e) \(\cos(u/2)\)
9. (15 points) Find a function which models the periodic behavior. The first minimum occurs at \( x = \frac{3}{4} \).

10. (15 points) Determine the area of a regular octagon with vertices on the unit circle.