

MATH 0031 - ALGEBRA
FINAL EXAM
SAMPLE FINAL EXAM

Exam length: 1 hour 50 minutes

INSTRUCTIONS:

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.
2. Show ALL of your calculations and display answers clearly. You may leave your final answers in exact form. UNJUSTIFIED ANSWERS will receive NO credit.
3. WRITE YOUR SOLUTIONS in the space provided. EXTRA SPACE is available on the BACKS of the pages. When using these back pages, clearly LABEL the problem, and also clearly indicate on the appropriate front page where your back-page solution (or continuation of a solution) is located.
4. Write neatly. Cross out any work that you do not wish to be considered for grading.
5. Academic Integrity Strictly Applies. Looking at another person's paper is reason to assume cheating and your paper will be taken.
6. All Cell phones and electronic devices must be OFF and put away and hats removed.

1. The points $A = (-1, 2)$ and $B = (7, -4)$ are at the ends of a diameter of a circle.

(a) (5 points) Find coordinates of the center of the circle C .

(b) (5 points) Find the equation of the circle.

(c) (5 points) Determine the equation of line passing through points A and B . Write it in the slope-intercept form.

(d) (5 points) Determine the equation of line perpendicular to the diameter AB which passes through the center C . Write it in the slope-intercept form.

2. (10 points) A book, a pen, and a notebook together cost \$100. The book costs \$80 more than the pen and the notebook costs twice as much as the pen. What is the price of each item? To solve the problem use one equation with one unknown.

3. Simplify. Write answers in the form $a+bi$ where a and b are real numbers.

(a) (5 points) $(5 - 3i) - (-1 + i)$

(b) (5 points) $\frac{-2 + i}{1 - i}$

(c) (5 points) i^{33}

4. Solve inequalities. Write answers in interval notation.

(a) (5 points) $x^2 + x > 2$.

(b) (5 points) $|3x + 2| > 2$.

(c) (5 points) $\frac{x}{x-1} \leq 1$.

5. Find all asymptotes of the graph of the function $f(x) = \frac{3x + 5}{x^2 - 2x - 3}$.

Support your answer. If the function doesn't have asymptotes of certain type(s) explain why.

(a) (5 points) Vertical.

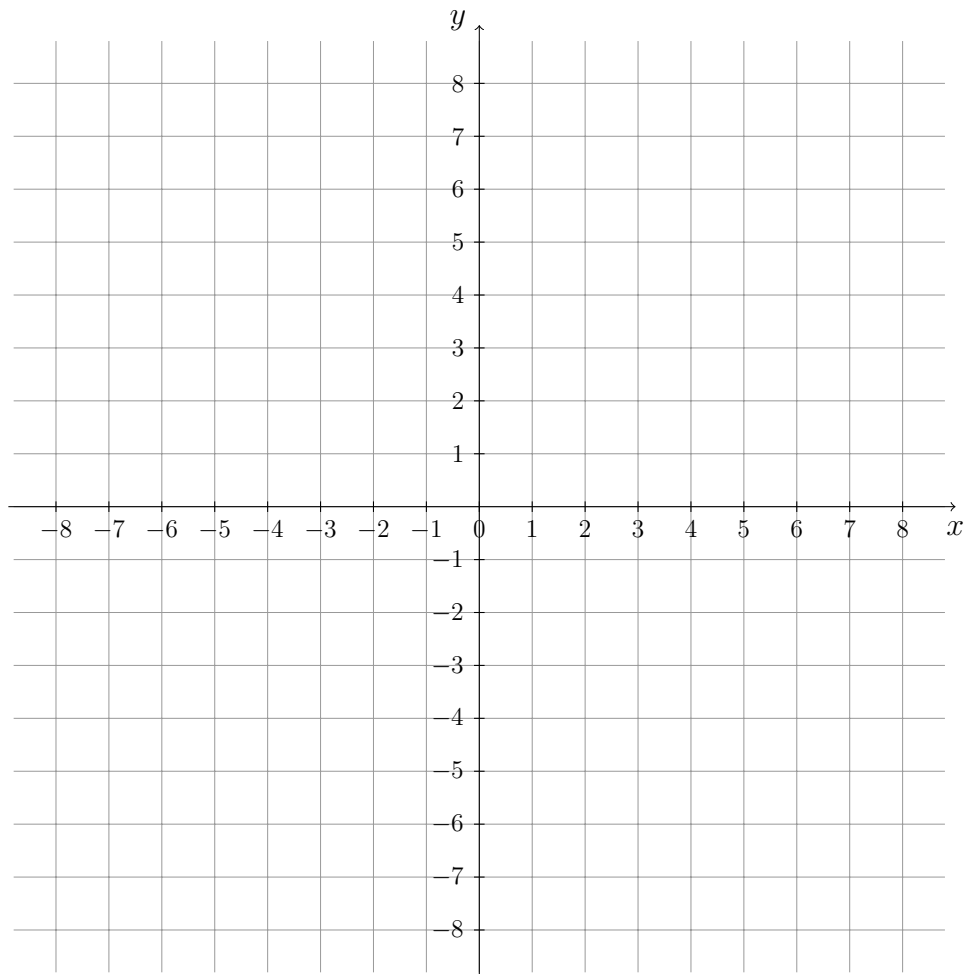
(b) (5 points) Horizontal.

(c) (5 points) Oblique.

6. $f(x) = x^2 - 6x + 7$.

(a) (5 points) Write $f(x)$ in the form $a(x - h)^2 + k$ by completing square. Find its vertex and axis of symmetry.

(b) (5 points) Sketch the graph of the function $f(x)$. Mark the vertex and draw the axis of symmetry.



7. Consider the polynomial function

$$h(x) = \frac{1}{4}x^3 + \frac{1}{2}x^2 - x - 2 = \frac{1}{4}(x + 2)(x^2 - 4).$$

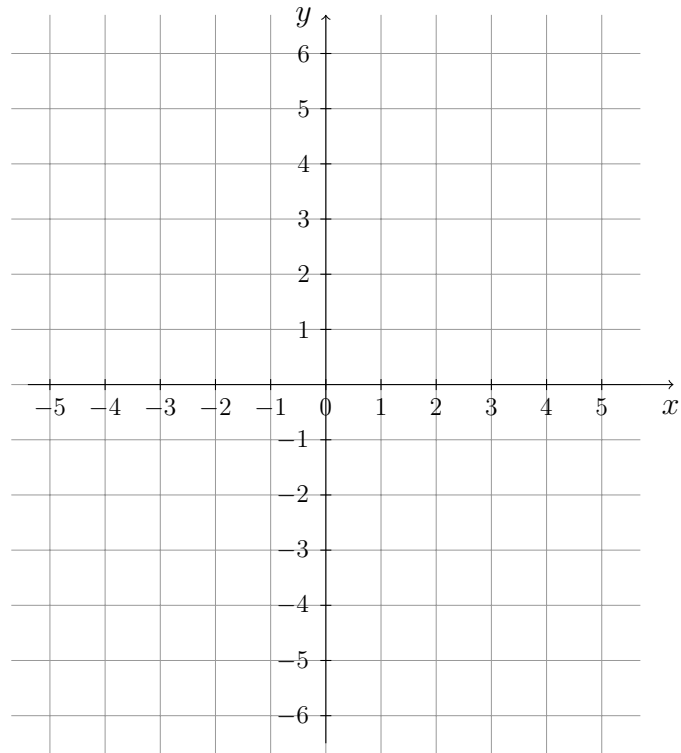
(a) (5 points) Find zeros of $h(x)$ and their multiplicities.

(b) (5 points) Zeros divide the x -axis into intervals. Determine a sign of $h(x)$ on each such interval.

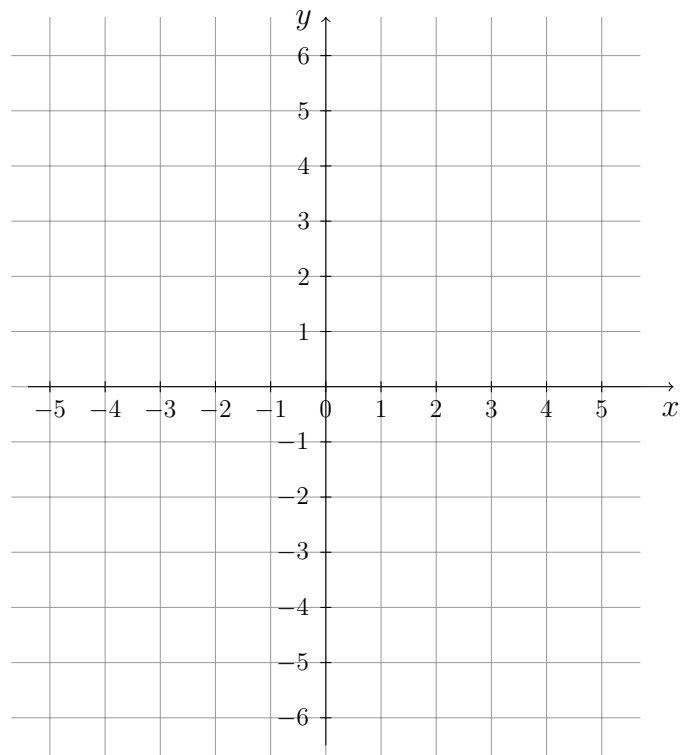
(c) (5 points) At each zero determine if the graph of $h(x)$ crosses the x -axis or is tangent to it.

(d) (5 points) Find the y -intercept of the graph of $h(x)$.

(e) (5 points) Sketch the graph of $h(x)$.



(f) (5 points) Using the graph of $h(x)$ sketch the graph of the function $f(x) = h(-x) - 1$.



8. $g(x) = x^3 - 5x^2 + x - 5.$

(a) (5 points) By using the rational zeros theorem find all possible rational zeros of $g(x)$.

(b) (5 points) $g(x)$ has only one rational zero. By using synthetic division find it.

(c) (5 points) Find all other zeros and factor $g(x)$.

9. (10 points) Find a polynomial function $f(x)$ of least degree with rational coefficients that has 2 and $\sqrt{5}$ as zeros.

10. (a) (5 points) The function $f(x) = 5^{4x-12}$ is one-to-one. Find a formula for the inverse function $f^{-1}(x)$.

(b) (5 points) Find the value of x for which $f^{-1}(x) = 3.5$.

11. Simplify

(a) (5 points) $\frac{\ln 9}{\ln 3}$

(b) (5 points) $\log_2 \frac{\sqrt[4]{x}}{y^2} - \log_2 \frac{x^{5/4}}{y^3}$

12. Solve equations

(a) (5 points) $3^{2x-5} = 27$

(b) (5 points) $\log_2(x-1) + \log_2 x = 1$

13. Magnesium-27 has a half-life of 10 minutes.

(a) (5 points) What is the exponential decay rate k for Magnesium-27 if time is measured in minutes?

(b) (5 points) A sample contains 100 grams of Magnesium-27. How much of Magnesium-27 is left in the sample after 20 minutes?

14. One day Mike bought two cups of coffee and two donuts for the cost of \$9. The next day he bought one cup of coffee and six donuts for a total of \$12. What is the cost of one cup of coffee and the cost of one donut?

(a) (5 points) Write a system of two equations with two unknowns that describes the problem.

(b) (5 points) Apply Cramer's rule to solve the system. For that calculate determinants of three 2×2 matrices.

15. (10 points) Graph $3x + 2y > 6$.

