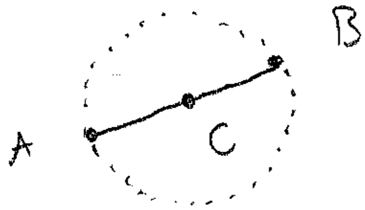
1. Given the points $A = (2,8)$ and $B = (10,14)$,



(a) (5pt) Determine the midpoint C.

$$C = \text{midpt}(A,B) = \left(\frac{2+10}{2}, \frac{8+14}{2}\right) = \left(\frac{12}{2}, \frac{22}{2}\right) = (6,11)$$

(b) (10pt) If the line segment connecting A and B is the diameter of a circle, determine the equation of the circle.

The equation for a circle is $(x-x_c)^2+(y-y_c)^2 = r^2$

where $(x_c, y_c) = (6, 11)$ is the center of the circle.

and the diameter is $d = \sqrt{(10-2)^2+(14-8)^2} = \sqrt{8^2+6^2} = \sqrt{100} = \boxed{10}$

Hence $r = \frac{10}{2} = 5 \Rightarrow \boxed{(x-6)^2+(y-11)^2 = 25}$

(c) (5pt) Determine the equation of line having points A and B.

$y - y_A = m(x - x_A)$. $m = \frac{14-8}{10-2} = \frac{6}{8} = \frac{3}{4}$.

$$\boxed{y - 8 = \frac{3}{4}(x-2)}$$

(d) (5pt) Determine the equation of the line perpendicular to the diameter AB which passes through the center of the circle.

$m^{\perp} = \frac{-4}{3}$ (negative reciprocal of $m = \frac{3}{4}$).

$$\boxed{y - 11 = \frac{-4}{3}(x-6)}$$

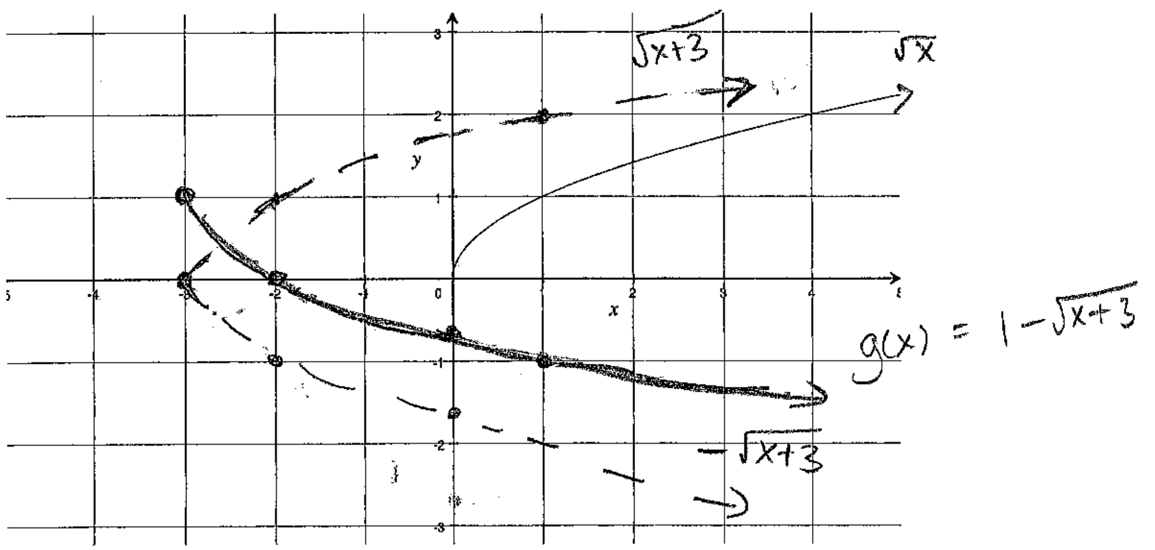
2. (10pt) For the function $f(x) = x^2 + 2x - 3$, construct and simplify the difference quotient $\dfrac{f(2+h) - f(2)}{h}$.

$$f(2+h) = (2+h)^2 + 2(2+h) - 3$$
$$= 4 + 2h + h^2 + 4 + 2h - 3$$
$$= 5 + 4h + h^2$$

$$f(2) = 2^2 + 2(2) - 3 = 8 - 3 = 5,$$

$$\frac{f(2+h) - f(2)}{h} = \frac{1}{h}(\cancel{5} + 4h + h^2 - \cancel{5}) = \frac{h}{h}(4+h) = \boxed{4+h}$$

3. (10 pt) The graph of $f(x) = \sqrt{x}$ is shown. On the same axis given, use transformations to sketch the graph of $g(x) = 1 - \sqrt{x+3}$



$\sqrt{x+3}$ (shift left 3-units)

$-\sqrt{x+3}$ (reflect across x-axis)

$1 - \sqrt{x+3}$ (shift up 1-unit).

4. (10pt) Using the Rational Root Theorem, list all possible rational solutions to the equation $2x^3 + 3x^2 - 11x - 6 = 0$, then find the actual solutions.

$$\frac{p}{q} = \frac{\pm 6, \pm 3, \pm 2, \pm 1}{\pm 2, \pm 1} = \pm 6, \pm 3, \pm 2, \pm \frac{3}{2}, \pm 1, \pm \frac{1}{2}.$$

$\underline{\text{Try}\quad x = 2}$

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -11 & -6 \\ & \downarrow & 4 & 14 & 6 \\ \hline & 2 & 7 & 3 & \boxed{0} \end{array}$$

$\Rightarrow 2x^3 + 3x^2 - 11x - 6 = (x-2)(2x^2+7x+3) = (x-2)(2x+1)(x+3)$

$\Rightarrow \boxed{x = 2, \frac{-1}{2}, -3}$

5. (5pt) Determine the polynomial function $f(x)$ of degree 4 with $x = -2$ and $x = 3$ zeros of multiplicity one and having $x = 1$ as a zero of multiplicity 2, such that $f(0) = \frac{1}{2}$.

$\deg(f) = 4$. $f(0) = \frac{1}{2}$.

Zeroes: $x = -2, 3$ (multiplicity 1).
$x = 1$ (multiplicity 2).

$$f(x) = a(x+2)(x-3)(x-1)^2$$

$$\frac{1}{2} = f(0) = a(2)(-3)(-1)^2 = -6a.$$

$$\Rightarrow \frac{1}{2} = -6a \Rightarrow a = \frac{-1}{12} \Rightarrow \boxed{f(x) = \frac{-1}{12}(x+2)(x-3)(x-1)^2}$$

6. (5pt each) Write in the form $a + bi$

(a) $(-3+7i)(4-5i)$ $= -12 + 15i + 28i - 35i^2$

$= 35 - 12 + 43i$

$= 23 + 43i.$

(b) $\dfrac{3+2i}{5+i}$

$$\frac{3+2i}{5+i} = \frac{(3+2i)(5-i)}{(5+i)(5-i)} = \frac{15-3i+10i-2i^2}{25+1} = \frac{17+7i}{26}$$

$$= \frac{17}{26} + \frac{7}{26} i$$

7. (5pt each) Solve the given equation.

(a) $x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0 \implies \boxed{x = -3} \text{ or } \boxed{x = 1}$

(b) $3t^2 + 4t - 2 = 0$

Quadratic Formula. $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, $a=3, b=4, c=-2$

$$= \frac{-4 \pm \sqrt{4^2-4(3)(-2)}}{2(3)} = \frac{-4 \pm \sqrt{4(4+6)}}{6}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{4}\sqrt{10}}{6}$$

$$= \boxed{-\frac{2}{3} \pm \frac{\sqrt{10}}{3}}$$

$$4x^2 + 15x + 8$$

$$(4x \quad)(x \quad)$$

8. (10pt) Determine all solutions to

$$\left(\sqrt{3x-1}\right)^2 = \left(2x+3\right)^2$$

$$3x - 1 = 4x^2 + 12x + 9$$

$$\Longleftrightarrow \quad 0 = 4x^2 + 9x + 10$$

Quadratic formula

$$\frac{-9 \pm \sqrt{9^2 - 4(4)(10)}}{2(4)} = \frac{-9 \pm \sqrt{79}\, i}{6}$$

$$= \boxed{-\frac{3}{2} \pm \frac{\sqrt{79}}{6} i}$$

9. (10pt) Determine all solutions to

$$(x+4)(x-4)\left[\frac{x}{x+4} + \frac{2}{x-4}\right] = 1 \cdot (x+4)(x-4)$$

$$x(x-4) + 2(x+4) = (x+4)(x-4)$$

$$\Rightarrow \quad x^2 - 4x + 2x + 8 = x^2 - 16$$

$$\Rightarrow \quad \cancel{x^2} - 2x + 8 = \cancel{x^2} - 16$$

$$\Longleftrightarrow \quad -2x = -24$$

$$\Longleftrightarrow \quad \boxed{x = 12}$$

Check!

$$LHS = \frac{12}{12+4} + \frac{2}{12-4}$$

$$= \frac{12}{16} + \frac{2}{8}$$

$$= \frac{3}{4} + \frac{1}{4} = \boxed{1}$$

$$RHS = \boxed{1}$$

10. (5pt) Solve the inequality:

$$|3x-2| \leq 8$$

$$\Longleftrightarrow \quad -8 \leq 3x - 2 \leq 8$$
$$\quad +2 \qquad\quad +2 \quad +2$$

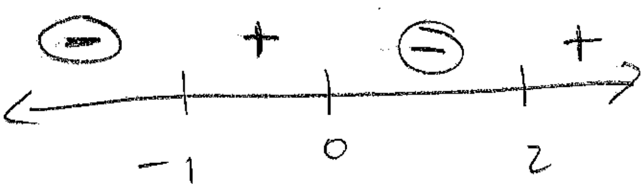
$$\Longleftrightarrow \quad -6 \leq 3x \leq 10$$

$$\Longleftrightarrow \quad \boxed{-2 \leq x \leq \frac{10}{3}}$$

11. (5pt) Solve the polynomial inequality:

$$x^2 + 3x + \cancel{2} \geq x^3 + x + \cancel{2}$$

$$\Longleftrightarrow \quad 0 \geq x^3 - x^2 - 2x = x(x^2 - x - 2)$$
$$= x(x-2)(x+1).$$



zeroes:
$x=0,\ x=2,\ x=-1$

$$\boxed{(-\infty, -1] \cup [0, 2]}$$

12. (5pt) Determine the inverse of the one-to-one function, $f(x) = \frac{2x+1}{x-3}$.

$y = f(x) = \frac{2x+1}{x-3}$. $\quad x \longleftrightarrow y \quad \Longrightarrow \quad x = \frac{2y+1}{y-3}$.

Solve for y:

$$x(y-3) = 2y + 1$$
$$\Longleftrightarrow \quad xy - 3x = 2y + 1$$
$$\Longleftrightarrow \quad xy - 2y = 3x + 1$$
$$\Longleftrightarrow \quad y(x-2) = 3x + 1$$
$$\Longleftrightarrow \quad y = \frac{3x+1}{x-2}$$

$$\boxed{f^{-1}(x) = \frac{3x+1}{x-2}}$$

$2^2 = 2 \cdot 2 = 4$

$2^3 = 2 \cdot 2 \cdot 2 = 8$

$2^4 = 16$

$2^5 = 32$.

13. (5pt) Simplify: $\log_2\left(\sqrt{32}\right)$ $= \log_2(32^{1/2}) = \frac{1}{2}\log_2(32)$

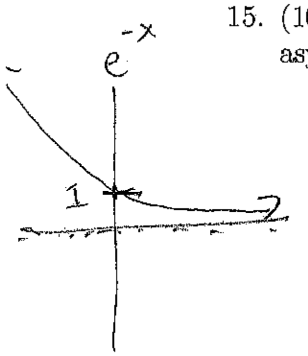
$= \frac{1}{2}\log_2(2)^5 = \frac{5}{2}\underbrace{\boxed{\log_2(2)}}_{}^{1} = \boxed{\frac{5}{2}}$.

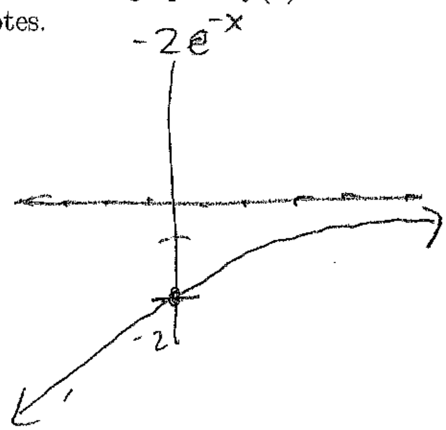
14. (10pt) If $\boxed{\ln(a) = 2.2,}$ and $\boxed{\ln(b) = 1.8}$ and $\boxed{\ln c = 1.4,}$ determine $\ln\left(\sqrt{\frac{a^2 b}{c}}\right)$

$$\ln\left(\frac{a^2 b}{c}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{a^2 b}{c}\right) = \frac{1}{2}\left[\ln(a^2 b) - \ln(c)\right]$$

$$= \frac{1}{2}\left[\ln(a^2) + \ln(b) - \ln c\right] = \frac{1}{2}\left[2\ln(a) + \ln(b) - \ln(c)\right]$$

15. (10pt) Sketch the graph of $f(x) = 10 - 2e^{-x}$. Mark carefully any intercepts and/or asymptotes.



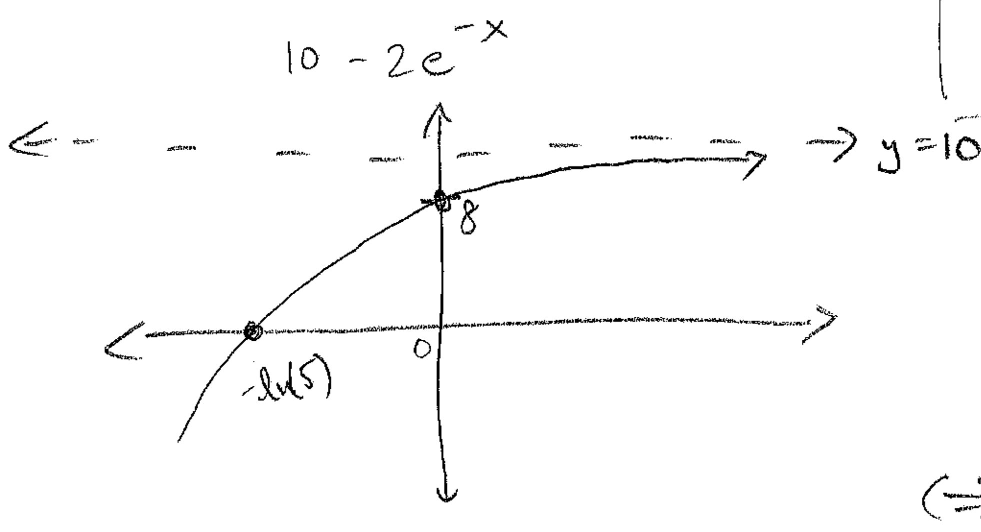


$$= \frac{1}{2}\left[2(2.2) + 1.8 - 1.4\right]$$

$$= \frac{1}{2}\left[4.4 + .4\right]$$

$$= \frac{1}{2}(4.8)$$

$$= \boxed{2.4}.$$



$f(0) = 10 - 2 = \boxed{8}$ y-intercept

$(0, 8)$

$0 = 10 - 2e^{-x}$

$\Leftrightarrow 5 = e^{-x} \Leftrightarrow \boxed{x = -\ln 5}$

16. (10pt) Determine all solutions:

$$\log_{10}(2x+1) - \log_{10}(x-2) = 1$$

$$\log_{10}\left(\frac{2x+1}{x-2}\right) = 1$$

$$\iff \frac{2x+1}{x-2} = 10$$

$$\iff 2x+1 = 10(x-2) = 10x - 20$$

$$\iff 21 = 8x \implies \boxed{x = \frac{21}{8}}$$

17. (10pt) Solve the linear system of equations

$$\begin{array}{lrcrcrcr} (1) & x & + & 2y & - & 3z & = & 1 \\ (2) & 2x & + & 3y & + & z & = & 6 \\ (3) & 3x & - & y & - & z & = & -10 \end{array}$$

Eliminate z in two eqns.

(2) + (3):

$$2x + 3y + z = 6$$
$$3x - y - z = -10$$
$$5x + 2y = -4$$

(1) + 3·(2):

$$x + 2y - 3z = 1$$
$$6x + 9y + 3z = 18$$
$$7x + 11y = 19.$$

$$\boxed{(-2, 3, 1)}$$

Substitution

$$2y = -4 - 5x$$
$$y = -2 - \frac{5}{2}x$$

$$7x + 11\left(-2 - \frac{5}{2}x\right) = 19$$
$$\frac{14}{2}x - \frac{55}{2}x - 22 = 19$$
$$-\frac{41}{2}x = 41$$
$$\implies \boxed{x = -2}$$

Back-sub

$$y = -2 - \frac{5}{2}(-2)$$
$$= -2 + 5$$
$$= 3 \qquad \boxed{y = 3}$$

(3)

$$z = 3(-2) - 3 + 10$$
$$= 1$$

18. (10pt) Sheila has a total of \$1000 invested in two different accounts. One account yields 2.4% annual interest and the other yields 2% annual interest. Her total annual interest is \$22.50. How much does she have in each account?

X - amount in A , y - amount in B.

(1) $(x + y = 1000)\ .024.$

(2) $.024x + .02y = 22.5$

Eq(2) − .024 Eq.(1)

$$\begin{aligned} .024x + .02y &= 22.5 \\ -(.024x + .024y &= 24) \end{aligned}$$

$-.004y = -1.5$

$y = \frac{1.5}{.004} = \frac{3 \cdot 1000}{2 \cdot 4} = 3 \cdot 125 = \boxed{375}$

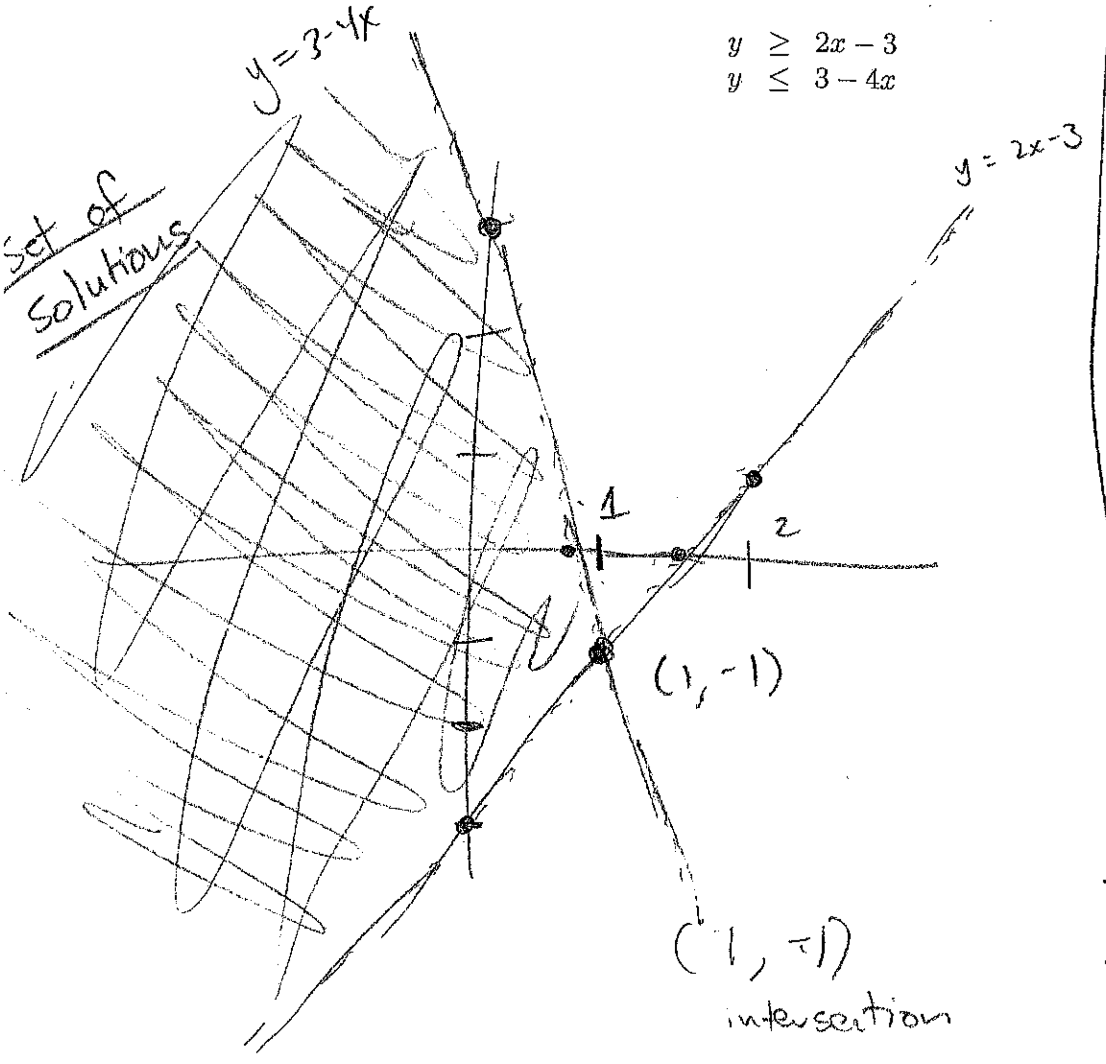
Backsub $\Rightarrow x = 1000 - 375 = \boxed{625}$

\$625 @ 2.4%

\$375 @ 2%

19. (15pt) Graph the system of inequalities and then find the coordinates of the vertex.

$$\begin{aligned} y &\geq 2x - 3 \\ y &\leq 3 - 4x \end{aligned}$$



$y = 2x - 3$

$-(y = 3 - 4x)$

$0 = -6 + 6x$

$\Rightarrow 6x = 6$

$\Rightarrow \boxed{x = 1}$

$\Rightarrow \boxed{y = -1}$

20. (15pt) Given the function $f(x) = \frac{x-1}{x^2-4}$, $= \frac{x-1}{(x-2)(x+2)}$

$p(x)$, $q(x)$

(a) Find and label all asymptotes

V.A. : $\boxed{x = \pm 2}$, H.A. : deg (p) < deg (q) $\Rightarrow$ $\boxed{y = 0}$

(b) Find the x and y intercepts.

y-intercept: $f(0) = \frac{1}{4}$, x-intercept

$0 = \frac{x-1}{x^2-4} \iff \boxed{x = 1}$

(c) Graph the function