Math 0031 **Sample Final Exam**

S o l u t i o n s

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1. Three points are given: $A = (-2, 2)$, $B = (2, 4)$, $C = (-4, 0)$

(a) (5 points) Find the midpoint D of the segment with endpoints B and C.

Solution: $D = \left(\dfrac{2 + (-4)}{2}, \dfrac{4 + 0}{2} \right)$ **+3 pts**

$D = (-1, 2)$ **+2 pts**

(b) (5 points) Write a slope-intercept equation of a line through A and B.

Solution: Slope is $m = \dfrac{4 - 2}{2 - (-2)} = \dfrac{2}{4} = \frac{1}{2}$ **+2 pts**

The line equation is $y - 2 = \frac{1}{2}(x - (-2))$, $y - 2 = \frac{1}{2}(x + 2)$, $y - 2 = \frac{1}{2}x + 1$ **+2 pts**

$y = \frac{1}{2}x + 3$ **+1 pts**

(c) (5 points) Write a slope-intercept equation of a line perpendicular to AB that passes through B.

Solution: Slope is $-\frac{1}{m} = -2$ **+2 pts**

The line equation is $y - 4 = -2(x - 2)$, $y - 4 = -2x + 4$ **+2 pts**

$y = -2x + 8$ **+1 pts**

(d) (5 points) Write an equation of a circle with center at A that passes through C.

Solution: The radius r of the circle is the length of AC:

$r^2 = (-4 - (-2))^2 + (0 - 2)^2 = (-2)^2 + (-2)^2 = 4 + 4 = 8$ **+2 pts**

The circle equation: $(x - (-2))^2 + (y - 2)^2 = 8$ **+2 pts**

$(x + 2)^2 + (y - 2)^2 = 8$ **+1 pts**

2. (10 points) The width of a rectangle is 4 in. greater than half of the length, and the perimeter is 44 in. Find the width and the length of the rectangle.

Solution: Let l be the length. Then the width is $w = \frac{1}{2}l + 4$. **+2 pts**

Then $2l + 2\left(\frac{1}{2}l + 4\right) = 44$ **+3 pts**

$3l + 8 = 40,\ 3l = 36,\ l = 12$ **+2 pts**

$w = \frac{1}{2} \cdot l + 4 = 6 + 4 = 10$ **+2 pts**

Answer: the width is 10 in and the length is 12 in. **+1 pts**

3. (10 points) With a \$70 membership of Nice Valley orchestra one can buy tickets for \$15 per a concert. Tickets for non members are \$20. For what number of concerts is it cheaper to buy tickets with the membership? Use an inequality to solve the problem.

Solution: A cost of x concerts with a membership is $70 + 15x$ and without a membership it is $20x$ dollars. **+3 pts**

We need to find when $70 + 15x < 20x$ **+2 pts**

$70 < 20x - 15x,\ 5x > 70$ **+2 pts**

$x > 14$ **+2 pts**

Answer:

It is cheaper to buy a membership if the number of concerts is greater than 14. **+1 pts**

4. Given that $f(x) = \sqrt{x+1}$ and $g(x) = 2x - 6$ find each of the following

(a) (5 points) $(f/g)(8)$.

Solution: $(f/g)(x) = \dfrac{\sqrt{x+1}}{2x-6}$ **+3 pts**

$(f/g)(8) = \dfrac{\sqrt{8+1}}{16-6} = \dfrac{3}{10} = 0.3$ **+2 pts**

(b) (5 points) Domain of the function $(f/g)(x)$ in interval notation.

Solution: $x + 1 \geq 0$ and $2x - 6 \neq 0$ **+2 pts**

$x \geq -1$ and $x \neq 3$ **+2 pts**

The domain is $[-1, 3) \cup (3, \infty)$ **+1 pts**

(c) (5 points) $(f \circ g)(15)$.

Solution: $(f \circ g)(x) = \sqrt{2x - 6 + 1} = \sqrt{2x - 5}$ **+3 pts**

$(f \circ g)(15) = \sqrt{30 - 5} = \sqrt{25} = 5$ **+2 pts**

(d) (5 points) $(g \circ f)(15)$.

Solution: $(g \circ f)(x) = 2\sqrt{x+1} - 6$ **+3 pts**

$(g \circ f)(15) = 2\sqrt{16} - 6 = 8 - 6 = 2$ **+2 pts**

(e) (5 points) value(s) of x such that $(g \circ f)(x) = 0$.

Solution: $2\sqrt{x+1} - 6 = 0$ **+1 pts**

$2\sqrt{x+1} = 6$ **+1 pts**

$\sqrt{x+1} = 3$ **+1 pts**

$x + 1 = 9$ **+1 pts**

$x = 8$ **+1 pts**

5. Determine whether the function is even, odd, or neither

(a) (5 points) $f(x) = x^2 - |x|$.

Solution: $f(-x) = (-x)^2 - |-x| = x^2 - |x| = f(x)$ **+4 pts**

The function is even. **+1 pts**

(b) (5 points) $f(x) = x^3 - |x|$.

Solution: $f(-x) = (-x)^3 - |-x| = -x^3 - |x|$ **+2 pts**

$f(-x) \neq f(x), \ f(-x) \neq -f(x)$ **+2 pts**

The function is neither even nor odd. **+1 pts**

6. Simplify. Write answers in the form $a + bi$, where a and b are real numbers.

(a) (5 points) $\sqrt{-49} - 4i^2 - 5i - \sqrt{49}$.

Solution: $\sqrt{-49} - 4i^2 - 5i - \sqrt{49} = 7i + 4 - 5i - 7.$ **+4 pts**

$= -3 + 2i$ **+1 pts**

(b) (5 points) $\dfrac{1+i}{1-i}$.

Solution: $\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i} \cdot \dfrac{1+i}{1+i}.$ **+2 pts**

$= \dfrac{(1+i)^2}{(1-i)(1+i)} = \dfrac{1+2i-1}{1+1}$ **+2 pts**

$= \dfrac{2i}{2} = i$ **+1 pts**

7. (10 points) List all roots (real and complex) of the function $f(x) = (x^2 - 4x + 8)(x^2 - 4x + 3)$

Solution: $(x^2 - 4x + 8)(x^2 - 4x + 3) = 0.$ **+2 pts**

$x^2 - 4x + 8 = 0,\ x^2 - 4x + 3 = 0.$ **+2 pts**

$x = \frac{1}{2}(4 \pm \sqrt{16 - 32}),\ x = \frac{1}{2}(4 \pm \sqrt{16 - 12}).$ **+2 pts**

$x = \frac{1}{2}(4 \pm 4i),\ x = \frac{1}{2}(4 \pm 2).$ **+2 pts**

$x_1 = 2 - 2i,\ x_2 = 2 + 2i,\ x_3 = 1,\ x_4 = 3.$ **+2 pts**

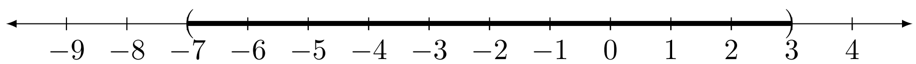
8. (10 points) Solve the inequality $|3x + 6| < 15$ and write interval notation for the solution set. Then graph the solution set.

Solution: $-15 < 3x + 6 < 15.$ **+2 pts**

$-21 < 3x < 9.$ **+2 pts**

$-7 < x < 3.$ **+2 pts**

The solution set is $\{x \mid -7 < x < 3\}$ or $(-7, 3)$. **+2 pts**



+2 pts

9. (10 points) Find the formula for the inverse function of $f(x) = \dfrac{3^{2x}+7}{4}$ and its domain.

Solution: $y = \dfrac{3^{2x}+7}{4}, \; x = \dfrac{3^{2y}+7}{4}$ **+2 pts**

$3^{2y} + 7 = 4x$ **+1 pts**

$3^{2y} = 4x - 7$ **+1 pts**

$2y = \log_3(4x-7)$ **+2 pts**

$y = \frac{1}{2}\log_3(4x-7)$ **+1 pts**

The inverse function is $f^{-1}(x) = \frac{1}{2}\log_3(4x-7)$ **+1 pts**

(Note $f^{-1}(x) = \frac{1}{2}\log_3|4x-7|$ is a wrong answer).

The domain is $4x - 7 > 0, \quad x > \frac{7}{4}$ or $\left(\frac{7}{4}, \infty\right)$ **+2 pts**

10. (10 points) Solve the inequality $\dfrac{x+2}{x^2-3x} > 0$

Solution: $x+2=0,\ x=-2$ **+1 pts**

$x^2-3x = x(x-3) = 0, \quad x=0,\ x=3$ **+1 pts**

Critical values are $x=-2$, $x=0$, and $x=3$ **+2 pts**

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,3)$	$(3,\infty)$
Test Value	$f(-3)=-\frac{1}{18}$	$f(-1)=\frac{1}{4}$	$f(1)=-\frac{3}{2}$	$f(4)=\frac{3}{2}$
Sign of $f(x)$	Negative	Positive	Negative	Positive

+4 pts

Answer: $(-2,0)\cup(3,\infty)$ **+2 pts**

11. In a certain year, a total of 4 million passengers took a cruise vacation. The global cruise industry has an exponential growth rate of 6% per year.

(a) (5 points) Find the exponential growth function.

Solution: $6\% = 0.06$, $P_0 = 4$ millions. **+2 pts**

$P(t) = 4e^{0.06t}$ **+3 pts**

(b) (5 points) In how many years the number of passengers will double? Leave your answer in exact form.

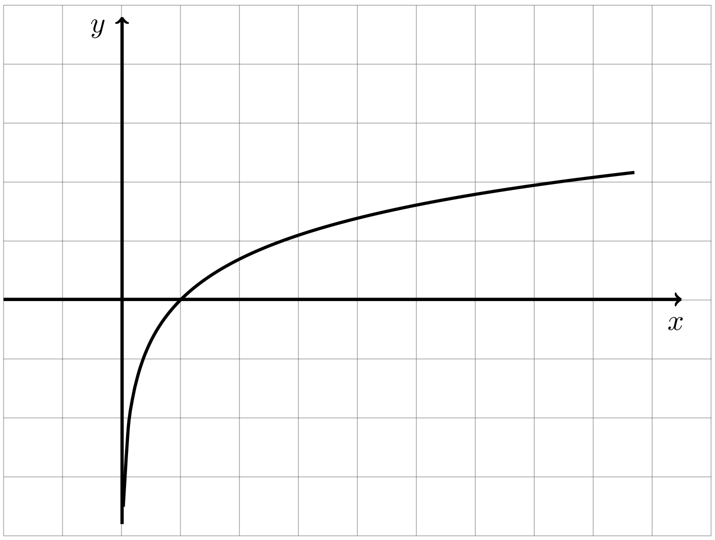
Solution: Let T be the doubling time. Then

$4e^{0.06T} = 2 \cdot 4,\ \ e^{0.06T} = 2$ **+2 pts**

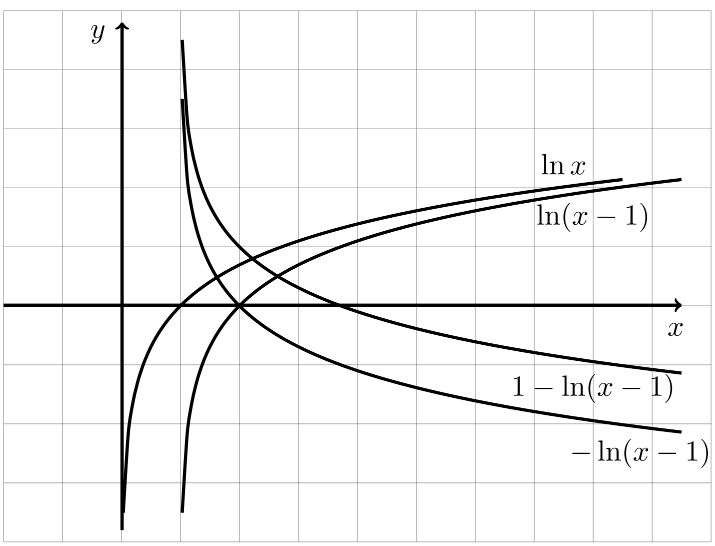
$\ln e^{0.06T} = \ln 2,\ \ 0.06T = \ln 2$ **+2 pts**

$T = \dfrac{\ln 2}{0.06}$ years. **+1 pts**

12. (15 points) The graph of $f(x) = \ln x$ is shown. On the same axis given, use transformations to sketch the graph of $g(x) = 1 - \ln(x-1)$. Describe how the graph of $g(x)$ was obtained from the graph of $f(x)$.



Solution:



Give **3 points** for every right plot of the functions $\ln(x-1)$, $\ln(x-1)$, and $1-\ln(x-1)$.

1. The graph of $\ln(x-1)$ is a shift of the graph $\ln x$ right one unit. **+2 pts**

2. The graph of $-\ln(x-1)$ is a reflection across the x-axis of the graph $\ln(x-1)$. **+2 pts**

3. The graph of $1-\ln(x-1)$ is a shift of the graph $-\ln(x-1)$ up one unit. **+2 pts**

13. (10 points) Simplify the equation $3\log_2 x + 4 - \log_2(8x^2) = 0$ and solve it for x.

Solution: $\log_2 \frac{(x^3)(2^4)}{8x^2} = \log_2 1$ **+4 pts**

$\log_2(2x) = \log_2 1$ **+3 pts**

$2x = 1$ **+2 pts**

$x = \frac{1}{2}$ **+1 pts**

14. The Coffee Shoppe sells a coffee blend made from two coffees, one costing \$5/lb and the other costing \$7/lb. The blended coffee sells for \$5.60/lb. The weight of the blended coffee is 100 lbs. Find how much of each coffee in pounds is used to obtain the desired blend.

(a) (5 points) Formulate the problem as a system of linear equations.

Solution: Denote by x the weight in lb of the first coffee in the blend and by y the weight of the second coffee. **+2 pts**

The cost of 100 lbs of the blended coffee is $100 \times 5.6 = 560$ dollars. **+1 pts**

Then we obtain a system of two equations with two unknowns:

$$\begin{aligned} x + \quad y &= 100 \\ 5x + 7y &= 560 \end{aligned}$$ **+2 pts**

(b) (5 points) Solve the system using the elimination method.

Solution: We multiply the first equation by -5

$$\begin{aligned} -5x - 5y &= -500 \\ 5x + 7y &= \quad 560 \end{aligned}$$ **+1 pts**

and add the result to the second equation:

$$2y = 60$$ **+1 pts**

Then

$$y = 30$$ **+1 pts**

and, from the first equation

$$x = 100 - y = 100 - 30 = 70$$ **+1 pts**

Answer: The weight of coffee that costs \$5/lb is 70 lbs

and the weight of coffee that costs \$7/lb is 30 lbs. **+1 pts**

15. For the given system of equations

$$\begin{aligned} x - 2y &= -1 \\ -2x + 5y &= 4 \end{aligned}$$

(a) (5 points) Write an equivalent matrix equation.

Solution: $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ **+5 pts**

(b) (5 points) Find an inverse matrix.

Solution: The matrix of the system is $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ **+1 pts**

The augmented matrix is

$$\left[\begin{array}{rr|rr} 1 & -2 & 1 & 0 \\ -2 & 5 & 1 & 0 \end{array}\right]$$ **+1 pts**

$$\left[\begin{array}{rr|rr} 1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right]$$ New row 2 = row 2 + 2 (row 1) **+1 pts**

$$\left[\begin{array}{rr|rr} 1 & 0 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{array}\right]$$ New row 1 = row 1 + 2 (row 2) **+1 pts**

Therefore, the inverse matrix is $A^{-1} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ **+1 pts**

(c) (5 points) Solve the system by using the inverse matrix.

Solution: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ **+1 pts**

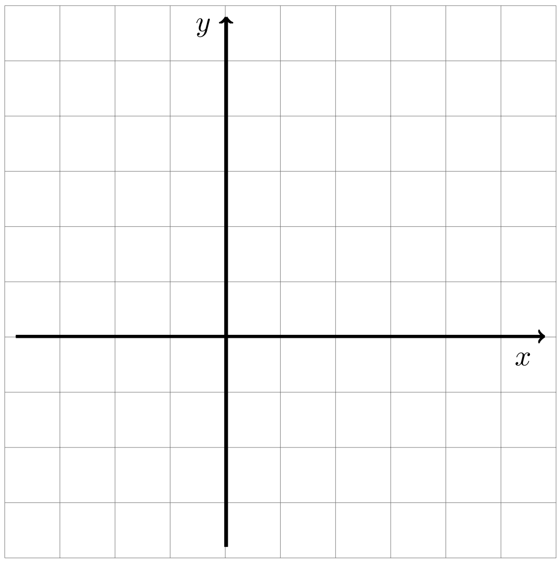
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 + 8 \\ -2 + 4 \end{bmatrix}$$ **+2 pts**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$ **+1 pts**

$x = 3, \ \ y = 2$ **+1 pts**

16. (15 points) Graph the solution set of the system.

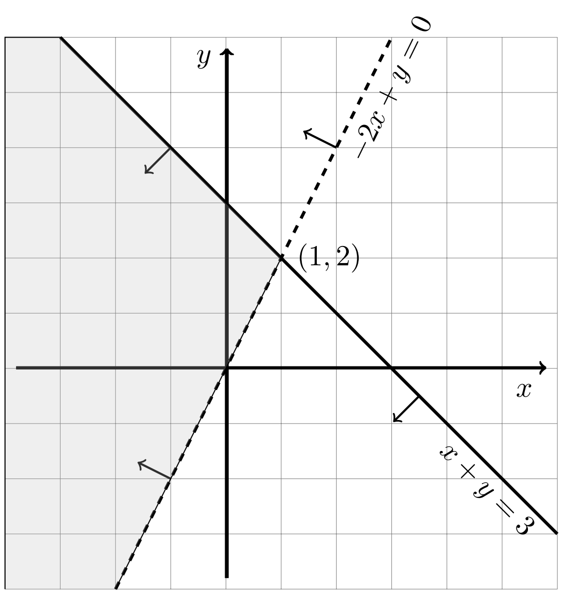
$$\begin{aligned} x + y &\geq 3 \\ -2x + y &> 0 \end{aligned}$$



Solution: The point of intersection of two curves:

$y = -x + 3 = 2x$, $3x = 3$, $x = 1$, $y = 2$, $(1, 2)$. **+3 pts**

The test points are $(0, 0)$ for $x + y \geq 3$ and $(0, 1)$ for $-2x + y > 0$. **+3 pts**



+9 pts