MATH 0031 - Algebra

SAMPLE FINAL EXAM

Exam length: 1 hour 50 minutes

INSTRUCTIONS:

- 1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.
- 2. Show ALL of your calculations and display answers clearly. You may leave your final answers in exact form. Unjustified answers will receive no credit.
- 3. WRITE YOUR SOLUTIONS in the space provided. EXTRA SPACE is available on the BACKS of the pages. When using these back pages, clearly LABEL the problem, and also clearly indicate on the appropriate front page where your back-page solution (or continuation of a solution) is located.
- 4. Write neatly. Cross out any work that you do not wish to be considered for grading.
- 5. Academic Integrity Strictly Applies. Looking at another person's paper is reason to assume cheating and your paper will be taken.
- 6. All Cell phones and electronic devices must be OFF and put away and hats removed.

- 1. (10 points) Given the points A = (-3, 7) and B = (-1, -5):
 - (a) Find the midpoint of A and B.

(b) Write the equation of the circle passing through the points A and B and centered at their midpoint.

(c) Find the equation of the line passing through the points A and B.

- **2.** (10 points) Let $f(x) = \sqrt{x}$, g(x) = 1 x, and h(x) = x 2.
 - (a) Write

$$k(x) = -2 + \sqrt{1-x} \,,$$

as a composition of f(x), g(x), and h(x).

(b) Given the graph of $f(x) = \sqrt{x}$, sketch the graph of k(x).



3. (5 points) Let $g(x) = x^2 - 3x + 8$. <u>Find</u> and <u>simplify</u> the difference quotient,

$$\frac{g(x+h) - g(x)}{h} \; .$$

4. (5 points)

(a) Solve

$$2t^2 - 3t + 4 = 0.$$

(b) Simplify

$$\frac{1-i}{6+2i}.$$

(c) Simplify

(3+4i)(2-i).

5. (10 points) For the graph of

$$f(x) = -x^2 + 2x + 8$$

- (a) Find the coordinates of the vertex.
- (b) Find the equation of the axis of symmetry.
- (c) What is the minimum or maximum value of f?
- (d) What is the range of f?
- (e) Sketch the graph of f.

- 6. (10 points) Let $f(x) = x^3 5x + 2$
 - (a) Find the leading coefficient and qualitatively describe the end behavior of f using the leading term test.

(b) Use the Intermediate Value Theorem to show f has a zero on the interval (0, 1). (no credit will be awarded if the Intermediate Value Theorem is not used).

(c) Verify by substitution that 2 is a zero of f(x) and then find its remaining zeroes.

(d) Find the remainder for when f(x) is divided by (x + 1).

7. (5 points) Solve the inequality: $x^2 + 3x + 1 \ge 8x + 15$.

- 8. (5 points) Suppose a 100 mg sample of an unknown substance decays radioactively so that after 10 years only 71 mg of the sample remain. Do not try to simplify answers as you are not permitted to use a calculator.
 - (a) What is the growth rate k?

(b) Find an equation which describes the amount A(t) remaining after t years.

(c) How many years will it take the sample to decay to 50 mg?

9. (10 points) Find the domain and all asymptotes of the rational function $f(x) = \frac{3x^2 - 5x + 1}{x - 1}$ then sketch a graph of f(x) below.

domain:

vertical asymptote:

oblique asymptote:



- **10.** (5 points) Let $f(x) = \frac{x+4}{x-3}$.
 - (a) Show that f(x) is **one-to-one**.
 - (b) Find the inverse function $f^{-1}(x)$.
 - (c) Find the range of f^{-1} .
 - (d) The graph of $f^{-1}(x)$ is a reflection of the graph of f(x) across what line?

11. (5 points) Solve (a)

$$4^{3x-5} = 16$$

(b) $2\ln x - \ln 5 = \ln(x+10)$

12. (5 points) Solve

13. (5 points) Compute the determinant

$$\begin{vmatrix} 2 & -1 & 4 \\ -3 & 1 & -2 \\ 5 & 3 & -1 \end{vmatrix}.$$

14. (10 points) For the matrices

$$A = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -5 & 1 \\ -2 & 4 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

either compute the following, or explain why it is not possible.

(a) AB

(b) BA

(c) A + B

(d) B + 2I

(e) B^{-1}