Ph.D. PRELIMINARY EXAMINATION

PART I – LINEAR ALGEBRA

January 10, 2001

1. Answer any 4 of the 6 questions.

2. Indicate below which 4 questions you wish to have graded. Do not indicate more than 4 questions for grading.

3. Use of soft lead (#2) pencil or a dark ink pen to record your answers on the answer sheets that have been provided.

4. Put your code number, but not your name, on each answer sheet that you submit. Confine your answers to the rectangular area indicated on the answer sheets.

CODE NUMBER: ________________

GRADE QUESTIONS: 1. _________ 2. _________ 3. _________

4. _________ 5. _________ 6. _________

Linear Algebra - January, 2001

1. Let V be an inner product space over $\mathbb{C}$, the complex field. Let T be a linear operator on V. If T is invertible show that T^* is invertible and $(T^*)^{-1} = (T^{-1})^*$.

2. Let A, B be $n \times n$ matrices over any field F. Show:

   (a) $trace(AB) = trace(BA)$.

   (b) All similar matrices have the same trace.

3. Let V be a finte dimensional real vector space and T is a linear operator on V. We say a subspace W is invariant under T if $TW \subseteq W$.

   Prove any T in $L(V, V)$ has an invariant subspace of dimension 1 or 2.

4. Let A, B be linear operators from V to V, where V is a finite dimensional vector space over $\mathbb{C}$. Let p be any polynomial such that $p(AB) = 0$.

   (a) Prove $q(x) = xp(x)$ satisfies $q(BA) = 0$.

   (b) Use this to show that either $m_{BA} = m_{AB}$ or $m_{AB} = xm_{BA}$ (where m_A is the minimal polynomial for A.

5. Let T be a linear operator on the complex vector space V. Let $\{\lambda_1 \cdots \lambda_k\}$ be the eigenvalues for T.

   Prove $|\lambda_i| < 1$ for all $i \Leftrightarrow T^n\alpha \to 0$ for all x in V.

6. Let V be a complex inner product space and let T be a linear operator on V.

   (a) Define T is self adjoint.

   (b) Prove T is self adjoint $\Leftrightarrow < T\alpha, \alpha >$ is real for all α.

   (c) Suppose $T^2 = T$ (i.e., T is idempotent). Show T is self adjoint $\Leftrightarrow T\,T^* = T^*T$.