Linear Algebra Preliminary Exam
April 2013

Problem 1 *Let A be a 4×4 real symmetric matrix. Suppose $A^2 + 3A = 0$ and $\operatorname{rank} A = 2$, find $\operatorname{tr} A$.*

Problem 2 *Suppose A and B are square complex matrices, such that AB is a projection. Is it true that BA is a projection? Prove of provide a counterexample.*

Problem 3 *Let A be a real symmetric positive definite matrix, show that*

$$A + A^{-1} - 2I$$

is semi positive definite.

Problem 4 *Let λ_i, $1 \leq i \leq m$ be m different eigenvalues of A, show that*

$$n(m-1) \leq \sum_{j=1}^{m} \operatorname{rank}(\lambda_j - A).$$

Problem 5 *Suppose A is a complex matrix satisfying $A^2 > 0$. Prove that A is diagonalizable.*

Problem 6 *Suppose A and B are normal complex $n \times n$ matrices. Prove that*

$$sr(AB) \leq sr(A)sr(B).$$

Here $sr(\cdot)$ is the spectral radius of a matrix.