Linear Algebra Preliminary Examinations, August 2011

1. Show that if Y and Z are subspaces of a finite-dimensional vector space then $(Y \cap Z)^{\perp} = Y^{\perp} + Z^{\perp}$ and $(Y + Z)^{\perp} = Y^{\perp} \cap Z^{\perp}$.

2. Suppose A is a complex $n \times n$ matrix, and $1 \leq m \leq n$. Prove that A has an invariant subspace of dimension m.

3. Suppose that V and W are complex inner product spaces. Suppose $A : V \to W$ and $B : V \to W$ are linear maps. Suppose $||A + B|| = ||A|| + ||B||$. Prove that $||A + 3B|| = ||A|| + 3||B|$.

4. Prove there are no two matrices $A, B \in C^{n \times n}$ that obey $AB - BA = I$.

5. Suppose P_1, P_2, and $P_1 + P_2$ are projections. Prove that $P_1 P_2 = 0$.

6. Prove or disprove.
Complex square matrices AA^* and A^*A are always unitarily similar.