1. Suppose that $U, V$ and $W$ are finite-dimensional linear spaces and that $S \in L(V,W)$ and $T \in L(U,V)$. Show that $\dim N_{ST} \leq \dim N_S + \dim N_T$.

2. Prove that the characteristic polynomial of a complex matrix $A$ divides its minimal polynomial if and only if all eigenspaces of $A$ are one-dimensional.

3. Let $Y$ be a subspace of (not necessarily finite-dimensional) linear space $X$. Show that $Y'$ (the dual of $Y$) is isomorphic to $X'/Y^\perp$.

4. For a given $n \times n$ real matrix $C$, consider a bilinear form $F_C$ on the space of real $n \times n$ matrices, defined by the formula $F_C(A,B) = \text{tr}(B^t \cdot C \cdot A)$. Prove that $F_C$ is non-degenerate if and only if $C$ is invertible.

5. Suppose $A^3$ is a unitary matrix. Prove that $A$ is diagonalizable.

6. Show that if $A$ is a linear map on a 2-dimensional complex Euclidean space and $\|A^2\| = \|A\|^2$ then $A$ is normal.