Linear Algebra Preliminary Examination
April 2010

Problem 1. Suppose A and B are $n \times n$ matrices and $AB = 0$. Prove that $\text{rank}A + \text{rank}B \leq n$.

Problem 2.
Suppose that P and Q are $n \times n$ matrices such that $P^2 = P$, $Q^2 = Q$, and $I - P - Q$ is invertible. Show that P and Q have the same rank.

Problem 3.
Suppose A and B are orthogonal projections in a finite-dimensional complex inner product space V. Suppose for some vector $x \in V$ we have $ABx = x$. Prove that $Ax = Bx = x$.

Problem 4.
Suppose A, B, and C are 2×2 matrices. Suppose $AB = BA$, $AC = CA$, and $BC \neq CB$. Prove that A is a scalar matrix.

Problem 5.
Let A and B be complex $n \times n$ matrices. Prove or disprove:

(a): If A and B are diagonalizable, so is AB.
(b): If $A^2 = A$, then A is diagonalizable.
(c): If A is invertible and A^2 is diagonalizable, then A is diagonalizable.

Problem 6.
Suppose A is a Hermitian matrix, and B is some matrix. Suppose $A^3B = BA^3$. Prove that $AB = BA$.