STABILITY CONSTANT OF THE WEAK*-FPP FOR DUAL OF SEPARABLE LINDENSTRAUSS SPACE

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ABSTRACT. Let X be a predual of ℓ_1 such that X^* has the w*-fpp (see [3]). We introduce two constants:

$$r^*(X) = \inf \left\{ r > 0 : (\operatorname{ext}(B_{\ell_1}))' \subset rB_{\ell_1} \right\}$$

$$\gamma^*(X) = \sup \left\{ \gamma \ge 1 : \text{ every } Y^* \text{ has } \sigma(Y^*, Y) \text{-fpp whenever } d(X, Y) \le \gamma \right\}.$$

It is well-known that if $r^*(X) = 0$, then $X = c_0$ and by results of Soardi [6] and Lim [5] we have $\gamma^*(c_0) = 2$. From Theorem 3.4 in [4] we know that if $r^*(X) = 1$, then $\gamma^*(X) = 1$. Further, if $r^*(X) \in (0,1)$ then the inequality $\gamma^*(X) \geq \frac{2}{1+r^*(X)}$ follows from the proof of Theorem 3.4 in [4]. We shall prove that if $r^*(X) \in (0,1)$, then $\gamma^*(X) \leq \frac{2}{1+r^*(X)}$.

References

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