ON G-HILBERT SCHEMES AND MCKAY CORRESPONDENCE

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Let G be a finite subgroup of $GL_n(\mathbb{C})$ and consider the quotient $X = \mathbb{C}^n/G$. In the famous paper of Bridgeland, King and Reid it was shown that in case G is contained in $SL_{2,3}(\mathbb{C})$, the G-Hilbert scheme (the fine moduli space of G-clusters in $\mathbb{C}^n$) is a crepant resolution of X. Moreover, one observes the following phenomenon: the number of nontrivial irreducible representations of G coincides with the number of irreducible components in the central fiber of the resolution. This usually goes under the name of McKay correspondence. The more modern formulation of McKay correspondence is the derived equivalence $D^G(\mathbb{C}^n) \to D(Y)$, where Y is a crepant resolution of X.

The plan for the talk is as follows. After giving the necessary definitions and explaining the McKay correspondence on the example of cyclic subgroups $\mathbb{Z}_n \subset SL_2(\mathbb{C})$, we state the results of the work in progress (joint with Timothy Logvinenko). Namely, that the G-Hilbert scheme for $\frac{1}{n}(1, \ldots, 1, -1, \ldots, -1)$ is smooth and the number of nontrivial irreducible representations of G coincides with the number of irreducible components in the central fiber of the resolution.