WENLONG PEI

Phone: (412)327-9248 Email: wep17@pitt.edu
Department of Mathematics, University of Pittsburgh, 615 Thackeray Hall, Pittsburgh, PA, 15260
Homepage: http://www.mathematics.pitt.edu/people/wenlong-pei

RESEARCH INTEREST

Primary:	Numerical Analysis	Computational Mathematics
	Computational Fluid Dynamics	Finite Difference Method
Secondary:	Mathematical Finance	

EDUCATION

University of Pittsburgh December 2021 or August 2022 (expected)
Ph.D. in Mathematics
Thesis: A Variable Time-stepping Algorithm for Flow Problems

University of Connecticut August 2014
M.S. in Applied Financial Mathematics

Shanghai University of International Business and Economics, China July 2011
Bachelor in Economics
Major: Finance

RESEARCH PAPER

1. Y. Qin, Y. Hou, W. Pei, and J. Li, *A variable time-stepping algorithm for the unsteady Stokes/Darcy model*, Journal of Computational and Applied Mathematics, page 113521, 2021. doi:10.1016/j.cam.2021.113521.
2. W. Layton, W. Pei, Y. Qin, and C. Trenchea, *Analysis of the variable step method of Dahlquist, Liniger and Nevanlinna for fluid flow*, Numerical Methods for Partial Differential Equations, (2021) doi.org/10.1002/num.22831.
3. W. Layton, W. Pei, and C. Trenchea, *Refactorization of a variable step, unconditionally stable method of Dahlquist, Liniger and Nevanlinna*, arXiv preprint arXiv:2108.09339, (2021). arxiv.org/abs/2108.09339
4. W. Layton, W. Pei, and C. Trenchea, *Some fine properties of the method of Dahlquist, Liniger and Nevanlinna*, (working in progress).
5. W. Layton, W. Pei, and C. Trenchea, *Semi-implicit algorithm of the method of Dahlquist, Liniger and Nevanlinna for Navier-Stokes equations*, (working in progress).
6. W. Layton, W. Pei, and C. Trenchea, *A variable time-stepping ensemble algorithm for fluid flow*, (working in progress).

CONFERENCE PRESENTATION

"Time accuracy of the variable step DLN method for Navier-Stokes equations", FEM Circus, Virginia Tech, Nov. 2019

"A variable time-stepping algorithm for the unsteady Stokes/Darcy model", FEM Circus, Online, April. 2021

TEACHING EXPERIENCE

Teaching Assistant for Recitations:

Math 120: Business Calculus	Fall 2016, Fall 2017
Math 220: Calculus I	Fall 2017, Spring 2018, Fall 2018
Math 230: Calculus II	Spring 2019, Summer 2019, Summer 2020
Math 240: Calculus III	Spring 2019, Summer 2019
Math 420: Introduction to Theory of One Variable Calculus	Spring 2021
Math 430: Introduction to Abstract Algebraic System	Fall 2021
Math 470: Actuarial Mathematics I	Spring 2018, Fall 2019, Spring 2020, Fall 2020
Math 1121: Actuarial Mathematics II	Spring 2020, Spring 2021
Math 1122: Actuarial Mathematics III	Fall 2020

PROGRAMMING SKILLS

Matlab, R, FreeFem++, C++, LaTeX

LAGRANGES

Chinese(Native) English(Fluent)

REFERENCE

Professor William Layton
Advisor
Department of Mathematics
University of Pittsburgh
wjl@pitt.edu
412-624-8312

Professor Catalin Trenchea
Co-advisor
Department of Mathematics
University of Pittsburgh
trenchea@pitt.edu
412-624-5681

Professor Ivan Yotov
Committee member
Department of Mathematics
University of Pittsburgh
yotov@pitt.edu
412-624-8338

Professor Sheng Xiong
Teaching reference
Department of Mathematics
University of Pittsburgh
sxiong@pitt.edu
412-624-2877

Research Statement: The Search for Time Accuracy in Computational Fluid Dynamics

Wenlong Pei

Email: wep17@pitt.edu

Computational fluid dynamics (CFD) is the subject devoted to predicting fluid transport, heat transfer, chemical reactions and related phenomena by numerically solving mathematical models that govern these processes. CFD techniques have been applied to various fields like meteorology, oceanography and astrophysics and in return CFD provides the ability to simulate any extreme condition in which conducting physical experiments is impossible. Additionally its cost are relatively low due to no use of expensive experiment equipments and meanwhile its execution time is short and likely to decrease as computers becomes more powerful.

Well-developed finite element methods for spacial discretization, accompanied by time discretization of low accuracy, are employed in most CFD simulations. Easily implemented time accurate algorithms with low storage, little explored but highly expected, would strengthen the reliability of CFD simulation. Time step adaptivity is the effective way of balancing time accuracy and computational efficiency, which results in great interest of variable time-stepping analysis for fluid problems. Dahlquist, Liniger and Nevanlinna [3] have proposed an one-parameter family, which is G-stable (nonlinearly, energetically stable) and second order accurate for any arbitrary sequence of time steps. To my knowledge, this method is the unique one that possesses such two excellent properties. **My work is to analyze the method of Dahlquist, Liniger and Nevanlinna (the DLN method), unearthing properties of the method and to apply it to fluid models**.

The DLN method (with parameter $\theta \in [0,1]$) for the ordinary differential system $y'(t) = f\left(t, y(t)\right)$ is

$$\sum_{\ell=0}^{2} \alpha_{\ell} y_{n-1+\ell} = (\alpha_2 k_n - \alpha_0 k_{n-1}) f\Big(\sum_{\ell=0}^{2} \beta_{\ell}^{(n)}(\varepsilon_n) t_{n-1+\ell}, \sum_{\ell=0}^{2} \beta_{\ell}^{(n)}(\varepsilon_n) y_{n-1+\ell} \Big),$$

where $k_n = t_{n+1} - t_n$ is the local time step, $\varepsilon_n = \frac{k_n - k_{n-1}}{k_n + k_{n-1}}$ is the stepsize variability and coefficients $\{\alpha_{\ell}, \beta_{\ell}^{(n)}(\varepsilon_n)\}_{\ell=0:2}$ are

$$
\begin{bmatrix}\alpha_2\\ \alpha_1\\ \alpha_0\end{bmatrix} = \begin{bmatrix}\frac{1}{2}(\theta+1)\\ -\theta\\ \frac{1}{2}(\theta-1)\end{bmatrix}, \qquad \begin{bmatrix}\beta_2^{(n)}(\varepsilon_n)\\ \beta_1^{(n)}(\varepsilon_n)\\ \beta_0^{(n)}(\varepsilon_n)\end{bmatrix} = \begin{bmatrix}\frac{1}{4}\left(1+\frac{1-\theta^2}{(1+\varepsilon_n\theta)^2}+\varepsilon_n^2\frac{\theta(1-\theta^2)}{(1+\varepsilon_n\theta)^2}+\theta\right)\\ \frac{1}{2}\left(1-\frac{1-\theta^2}{(1+\varepsilon_n\theta)^2}\right)\\ \frac{1}{4}\left(1+\frac{1-\theta^2}{(1+\varepsilon_n\theta)^2}-\varepsilon_n^2\frac{\theta(1-\theta^2)}{(1+\varepsilon_n\theta)^2}-\theta\right)\end{bmatrix}.
$$

Research Achievements

The complicated form of the DLN method deters its testing in CFD where its excellent properties should be valued. To solve this issue, I refactorize the DLN method by adding pre-filter and post-filter on backward Euler method and obtain the following algorithm for each step computation [7]

Algorithm: Refactorization of the DLN Method

Input: y_n, y_{n-1} ;

$y_n^{\text{old}} \Leftarrow a_1^{(n)} y_n + a_0^{(n)} y_{n-1}$;

$k_n^{\text{new}} \Leftarrow b^{(n)} \widehat{k}_n$;

$t_{n+1}^{\text{new}} \Leftarrow \beta_2^{(n)} t_{n+1} + \beta_1^{(n)} t_n + \beta_0^{(n)} t_{n-1}$; `// pre-filter for` y_n `,` k_n `and` t_{n+1}

Solve for y_{n+1}^{temp}: $\frac{y_{n+1}^{\text{temp}} - y_n^{\text{old}}}{k_n^{\text{new}}} = f\left(t_{n+1}^{\text{new}}, y_{n+1}^{\text{temp}}\right)$; `// backward Euler algorithm`

$y_{n+1} \Leftarrow y_{n+1}^{\text{temp}} + \left(c_2^{(n)} y_{n+1}^{\text{temp}} + c_1^{(n)} y_n + c_0^{(n)} y_{n-1}\right)$; `// post-filter for` $y_{n+1}^{\texttt{temp}}$

To further develop the DLN method, I have obtained the expressions of numerical dissipation in G-stability identity and local truncation error (LTE) for the DLN algorithm, two important criteria for measuring effect of time-stepping algorithms on fluid models. For constant step case, I have proved L-stability of the DLN method. To adapt time steps, I extend Gresho's idea [2] to derive some estimators of LTE for variable time-stepping DLN method with aid of some explicit methods and the general time step controller proposed by Söderlind [11].

Equipped with these detailed properties, **I have applied the variable time-stepping DLN method to some flow problems (unsteady Stokes/Darcy model and Navier Stokes equations (NSE)) and performed a completed stability and error analysis of approximate solutions** [6, 10]. The approximate solutions are unconditionally, long time stable and second order accurate under variable time steps. I have implemented DLN method to Taylor-Green benchmark problem [12] to confirm the second order convergence rate and adjusted time step using the minimum-dissipation criteria of Capuano, Sanderse, De Angelis and Coppola [1] for the variable step test problem from Jiang [4] that is inspired by flow between offset cylinders. The minimum-dissipation strategy, adding only a few lines of code, can be implemented simply to suppress the time-integration error with desired tolerance and increase efficiency dramatically.

Current Work and Future Plan

To solve J ($J > 1$) NSEs simultaneously, Jiang and Layton [4] combine backward Euler method and ensemble averaging technique to obtain the following algorithm for each jth NSE ($j = 1, 2, \cdots, J$)

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + \langle u^n \rangle \cdot \nabla u_j^{n+1} + \left(u_j^n - \langle u^n \rangle\right) \cdot \nabla u_j^n - \nu \Delta u_j^{n+1} + \nabla p_j^{n+1} = f_j^{n+1},$$
$$\nabla \cdot u_j^{n+1} = 0,$$

where $\langle u^n \rangle := \frac{1}{J}\left(\sum_{j=1}^{J} u_j^n\right)$ is the ensemble average. The above algorithm at each step is equivalent to the following block linear system

$$\begin{bmatrix} \frac{1}{\Delta t} M_u + \nu S_u + N_u\left(\langle u^n \rangle\right) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} u_j^{n+1} \\ p_j^{n+1} \end{bmatrix} = \begin{bmatrix} f_j^{n+1} + \left(\frac{1}{\Delta t} M_u + N_u\left(u_j^n - \langle u^n \rangle\right)\right) u_j^n \\ 0 \end{bmatrix}, \tag{1}$$

where M_u is the mass matrix, S_u is the diffusion matrix, N_u is the convection matrix and B is the continuity matrix. The resulting coefficient matrix in (1), denoted by A, is independent of j. Denote the solution vector and the vector on the right hand side by x_j and b_j respectively, the ensemble algorithms of J NSE is reduced to

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 | x_2 | \cdots | x_J \end{bmatrix} = \begin{bmatrix} b_1 | b_2 | \cdots | b_J \end{bmatrix},$$

which solves J NSE at the same time as well as significantly reduces the storage due to the shared coefficient matrix. **Now I have combined the DLN method with ensemble algorithm for NSE and completed the stability and error analysis** [8].

The artificial compression algorithm [5] for NSE, changing mass conservation equation $\nabla \cdot u = 0$ a little by $\delta p_t + \nabla \cdot u = 0$ ($0 < \delta \ll 1$), solves pressure explicitly and largely reduce computation cost in another way. **I have tried the DLN-artificial compression algorithm (the DLN-AC algorithm) to NSE and proved the stability and convergence of approximate solutions under constant δ** [9].

My future research plan includes

- **For DLN algorithm on NSE**: I will apply semi-implicit DLN algorithm to NSEs for stability and convergence analysis and then try numerical simulations with adaptivity by LTE criteria.
- **For DLN-ensemble algorithm on NSE**: I will try simulations of variable time-stepping DLN-ensemble algorithm on NSE (adapting time step) and then apply DLN-ensemble algorithm to eddy viscosity model.
- **For DLN-AC algorithm on NSE**: I will try stability and error analysis of DLN-AC algorithm on NSE with variable δ and then simulate some classic test problems with both constant δ and variable δ.

References

[1] F. Capuano, B. Sanderse, E. D. Angelis, and G. Coppola, *A minimum-dissipation time-integration strategy for large-eddy simulation of incompressible turbulent flows*, 2017.

[2] C. Chao, S. Orszag, and W. Shyy, *Recent Advances in Computational Fluid Dynamics: Proceedings of the US/ROC (Taiwan) Joint Workshop on Recent Advances in Computational Fluid Dynamics*, Lecture Notes in Engineering, Springer Berlin Heidelberg, 2013.

[3] G. G. Dahlquist, W. Liniger, and O. Nevanlinna, *Stability of two-step methods for variable integration steps*, SIAM Journal on Numerical Analysis, 20 (1983), pp. 1071–1085.

[4] N. Jiang and W. Layton, *An algorithm for fast calculation of flow ensembles*, Int. J. Uncertain. Quantif, 4 (2014), pp. 273–301.

[5] W. Layton and M. McLaughlin, *Doubly-adaptive artificial compression methods for incompressible flow*, arXiv e-prints, (2019), p. arXiv:1907.08235.

[6] W. Layton, **W. PEI**, Y. Qin, and C. Trenchea, *Analysis of the variable step method of Dahlquist, Liniger and Nevanlinna for fluid flow*, Numerical Methods for Partial Differential Equations, accepted (2021).

[7] W. Layton, **W. PEI**, and C. Trenchea, *Refactorization of a variable step, unconditionally stable method of Dahlquist, Liniger and Nevanlinna*, arXiv preprint arXiv:2108.09339, (2021).

[8] W. Layton, **W. PEI**, and C. Trenchea, *A variable time-stepping ensemble algorithm for fluid flow*, (in preparation).

[9] W. Layton, **W. PEI**, and C. Trenchea, *The DLN artificial compression algorithm for Navier Stokes equations*, (in preparation).

[10] Y. Qin, Y. Hou, **W. PEI**, and J. Li, *A variable time-stepping algorithm for the unsteady Stokes/Darcy model*, Journal of Computational and Applied Mathematics, page 113521, 2021.

[11] G. Söderlind, *Digital filters in adaptive time-stepping*, ACM Transactions on Mathematical Software (TOMS), 29 (2003), pp. 1–26.

[12] M. Van Dyke, *An album of fluid motion*, (1982).

Teaching Statement: Knowledge, Enthusiasm, Patience and the Desire to Help Others

Wenlong Pei

Teaching is of great joy to me and it comes naturally to me. I am sixth-year Ph.D. student at the Department of Mathematics, University of Pittsburgh and I worked as teaching assistant every semester. During the past five years, I have accumulated a wide variety of teaching experience by teaching recitations for different level of undergraduate courses, which gives me both challenge and pleasure. I feel so honored to cultivate students with mathematical skills of solving problems and the way of logical thinking. Additionally, the most important lesson I have learned are qualities for an eligible teacher: knowledge, enthusiasm, patience and the desire to help others.

I started my teaching experience as TA for Business Calculus recitation which is designed for undergraduate students of business majors. The aim of this course is to provide students with required mathematical materials(definitions, formulas and theorems) as well as to connect mathematics to economics and business analysis, giving students the belief that mathematics is really useful in economics and business. During the second year of Ph.D. study, I started to teach Calculus I,II,III. Most of my students were from science or engineering departments. These courses have higher requirements than Business Calculus and are used to endow students with necessary mathematical knowledge for further study in science and engineering. Due to my previous background in business, I worked as TA for recitations of actuarial mathematics courses when I became senior graduate student. Actuarial mathematics courses are high level courses for undergraduate students. The intension of recitations is to consolidate the knowledge offered in lectures by showing concrete examples and help students majoring actuarial science to prepare and pass Society of Actuaries (SOA) exams.

My duties include preparing recitation notes, solving problems before students, conducting quizzes biweekly, grading homework and holding office hours weekly. When I write recitation notes, I follow the syllabus closely and try to choose relatively challenging examples that would most likely simulate students' interest in mathematics and arouse their potential of conquering difficulties. Moreover, I have tried to select many practical problems according to diverse backgrounds and needs of students. For instance, there are many application problems dealing with business issues in Business Calculus recitation meanwhile problems of physics and engineering topics appear in recitations of Calculus I,II,III more often. In actuarial mathematics recitations, all examples I discuss in class come from previous SOA exams and requires quite familiarity with course contents and proficient problem-solving skills. Before every recitation, I write recitation notes (including problems and my solutions) clearly and put them into teaching system online. My students appreciate it a lot because they can preview them before my recitation and concentrate on my explanation without taking notes. For each specific example in class, I ask enlightening questions, which encourages my students to participate and think independently. If someone proposes the right way, I write down the steps and continue to ask what we should do next. Otherwise I give them hints so that they can advance. If there is a quiz in the recitation, I help students to summarize and review the contents needed for quiz. For homework grading, I correct wrong solutions from students with detailed comments.

Due to considerable efforts on TA work, I have shown my success in my teaching career and received numerous positive feedbacks from my students. Here are some selected comments.

- **on Business Calculus recitation:** *"Wenlong always worked through examples in class and always had time to answer people's questions."*
- **on Calculus II recitation:** *"He was always ready for recitation, and could easily explain how to do the problems. He was understanding with being online. Practice problems were always solved correctly and was very good with time management of the time of the class. Very good recitation."*
- **on Actuarial Mathematics I recitation:** *"clearly explanation of problems very prepared"*
- **on Actuarial Mathematics III recitation:** *"He chooses great problems to go over. His explanations are pretty clear."*

Through ten semesters of teaching in graduate school, I enjoy imparting knowledge to students from diverse backgrounds and helping them overcome academic hurdles. I have learned a lot from my past significant teaching experience and I am so proud of having so many teaching opportunities in University of Pittsburgh. I really hope that in the near future I can continue my teaching career.